
The Thermodynamic Properties of Fluid Helium

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THE THERMODYNAMIC PROPERTIES OF FLUID HELIUM

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Measurements have been made from which all the thermodynamic properties of fluid helium can be calculated in the temperature range from 3 to 20 °K and up to 100 atm pressure. The quantities measured were:

- (i) the specific heat at constant volume as a function of temperature and density,
- (ii) the pressure coefficient at constant volume $(\partial p/\partial T)_V$ also as a function of temperature and density,
- (iii) the pressure as a function of temperature at constant volume (isochores) for a range of densities.

A particular feature of the experiments is that the important derivative $(\partial p/\partial T)_V$, from which the changes of entropy and internal energy with volume at constant temperature are calculated, was measured directly by a differential method.

Starting from the known entropy and internal energy of the liquid near its normal boiling point, these two quantities were calculated for all single phase states within the experimental range. From these, and using the equation of state data, the enthalpy, free energy, Gibbs function, and the specific heat at constant pressure have been deduced. The thermodynamic functions, together with some useful state properties, are tabulated as functions of temperature and either volume or pressure as relevant.

The choice of the measured quantities was such that all the thermodynamic functions except the specific heat at constant pressure were obtained by integration of the experimental data; these functions therefore have the same accuracy as the measured quantities, about 1%.

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1. INTRODUCTION

The thermodynamic properties of fluid helium are of considerable theoretical and practical interest, and have been the subject of a number of investigations in the past. There are, however, surprisingly few experimental data on which calculations can be based, and most of the existing thermodynamic data have been obtained by piecing together measurements on a variety of properties made in different laboratories. In particular, no complete experimental survey of the technically important range from 3 to 20 °K and up to 100 atm pressure has ever been made. The present paper is intended to fill this gap in our knowledge.

The thermodynamic properties of a single phase of a substance can be determined in a number of ways which may be divided into two main classes: those in which equation of state data are used to calculate the difference of the properties of the real substance and those of the corresponding ideal gas, and those in which both thermal and state data are used to determine the properties directly. In the past, methods belonging to the first class have largely been used, not so much because of any inherent advantage as because the necessary thermal data did not exist.

Most of the experimental information on the equation of state in this region is in the form of isotherms, but isochores or isobars would be preferable for thermodynamic calculations. This may be seen by considering how changes of entropy S and internal energy U with volume are obtained from the equation of state. The link is provided by the elementary relations:

$$\begin{aligned} \left(\frac{\partial S}{\partial V}\right)_T &= \left(\frac{\partial p}{\partial T}\right)_V, \\ \left(\frac{\partial U}{\partial V}\right)_T &= T\left(\frac{\partial p}{\partial T}\right)_V - p. \end{aligned}$$

It is clear that $(\partial p/\partial T)_V$, which derives from isochores, is the important property. Similarly, when the pressure is taken as the independent variable, the quantity of interest is $(\partial V/\partial T)_p$, which is most directly obtained from isobars. It is equally clear from these equations that the accuracy with which thermodynamic quantities can be calculated is essentially that of the relevant derivative, which may be substantially less than the accuracy of p , V and T .

The changes of thermodynamic functions with temperature at constant volume or pressure are immediately related to the appropriate specific heat:

$$\begin{aligned} \left(\frac{\partial S}{\partial T}\right)_V &= \frac{C_V}{T}, \\ \left(\frac{\partial U}{\partial T}\right)_V &= C_V, \quad \text{etc.} \end{aligned}$$

Since all other thermodynamic functions can be calculated from the internal energy and entropy without differentiation, it appears that the most satisfactory type of experiment is one in which either C_V and $(\partial p/\partial T)_V$ or C_p and $(\partial V/\partial T)_p$ are measured. It is not strictly necessary to measure both parameters throughout the range of the variables, but it is advantageous to do so, since the internal consistency of the data can then be checked quite rigorously. The present experiments were designed accordingly, and the volume (actually the density) was chosen as the independent variable in preference to the pressure, purely for

reasons of convenience. In this temperature range, constant volume experiments are generally easier to perform than constant pressure ones, in direct contrast to the position at higher temperatures. A particular feature of these experiments was that the derivative $(\partial p/\partial T)_V$ was measured directly by a differential method, not by differentiating state data along isochores.

The difference between the present approach and those used by earlier investigators is best shown by summarizing the previous work in this field.

(i) The first entropy diagram was that of Keesom & Houthoff (1928). Due to the lack of reliable experimental data, this diagram was not very reliable; for instance, the inversion of the Joule–Thomson effect (Kapitza 1934) is not reflected in it.

(ii) The work of Zelmanov (1944*a*) covering the temperature range from 3 to 20 °K and up to pressures of 60 atm made use of his own measurements of the Joule–Thomson coefficient between 6 and 20 °K (Zelmanov 1940) and of enthalpy changes with temperature at 1 atm pressure (Zelmanov 1944*b*). Entropy values for the liquid at 4.2 °K and below were taken from the measurements of Keesom & Keesom (1933*a*). In the absence of other information, entropies for the range from 4.2 to 8 °K had to be obtained by direct graphical interpolation on the T - S diagram. The values are stated to be accurate to about 3%.

(iii) The entropy diagram of Akin (1950) does not extend below 10 °K, and the low-temperature region is not adequately represented. It is not therefore necessary to consider this diagram further.

(iv) The most recent diagram is that of Keesom, Bijl & Monté (1953) for the range of temperature and density from 1 to 500 °K and 0 to 700 Amagat units. It is evident from the wide scope of this diagram that very many experimental results have been used in its construction. In general, the thermodynamic functions were calculated from the equation of state using isothermal data. Between 4.2 and 15 °K no isotherms were available, and below 4.2 °K they extended only up to pressures of a few atmospheres.

The paucity of the data available to the older authors and the need for new information to supplement it are both evident.

Units

Density (ρ) was measured in absolute units but is expressed as a multiple of the critical density. The value adopted for this was 0.0688 g cm⁻³, and the same number must of course be used if it is desired to convert densities back to absolute densities. The most recent determinations of this parameter (Kerr 1957; Berman & Mate 1958) suggest that the adopted value is a little high, but this is of no great importance here as the number is used solely as a reducing parameter. The following relations are useful for converting to other units:

$$\text{density in Amagat units} = 385.4\rho,$$

$$\text{molar volume} = 58.18/\rho \text{ cm}^3.$$

Energy: all quantities having the dimensions of energy are expressed in joules. The relation 1 J = 9.8693 atm cm³ has been used extensively.

Pressure: the unit is the normal atmosphere = 1.01325×10^6 dyn cm⁻².

Quantity: all extensive quantities are molar, and are denoted by capital letters.

2. APPARATUS AND GENERAL TECHNIQUES

(a) *The cryostat*

The main requirement the apparatus had to fulfil was to provide a heat sink, large enough to cool about 1 mole of helium gas from room temperature down to 2°K , and to remain at helium temperatures for 10 to 15 h so that long experiments could be carried out.

Figure 1 is a schematic drawing of the cryostat. The outer Dewar vessel *I* was filled with liquid oxygen and the inner one *J* with liquid helium through the siphon *F*. About 1 l. of

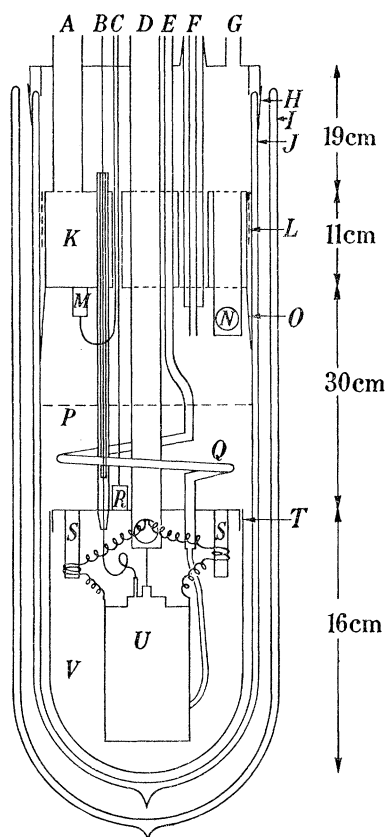


FIGURE 1. The cryostat.

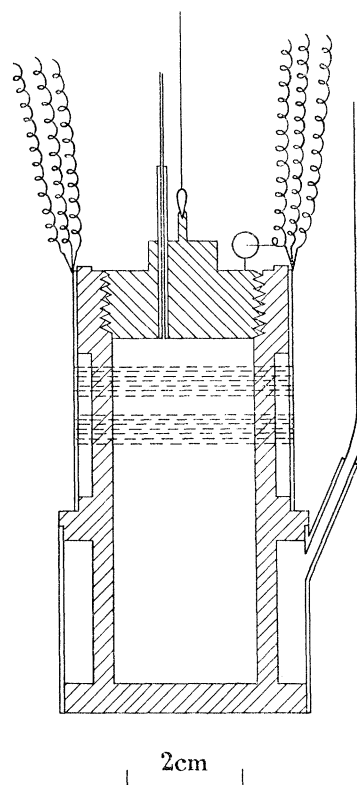


FIGURE 2. The calorimeter.

liquid helium was needed to bring the level up to *P*. The Dewar was closed by a rubber seal *H* at the top. Evaporating helium returned to a gas-holder through *N* to the line *G*. The brass can *K* was filled with liquid hydrogen through the german-silver tube *A*. This can served three purposes: first, it cooled the whole cryostat down to about 21°K before liquid helium was introduced; secondly, it provided a temperature bath for condensing liquid hydrogen to the calorimeter pot (§ 2 (b)) for thermometer calibrations; and thirdly, it reduced considerably the heat influx to the helium bath along the tubes and the inner wall of the Dewar. The temperature in this can was indicated by a simple gas thermometer consisting of the bulb *M* and one of the tubes *C*. Thermal contact between the can and the Dewar was made by a copper skirt *O* and a bandage *L* soaked in glycerin.

The sample tube *B* was closed immediately above the top cap of the cryostat by a needle valve having no dead volume on the calorimeter side. This tube was a german-silver

capillary of 0.3 mm inside diameter. It passed through the hydrogen can and the vacuum case V to the calorimeter U . Since the calorimeter could not be isolated from this tube, the amount of gas in it had to be made small compared with that in the calorimeter. To make this ratio favourable, as much as possible of the tube was kept relatively warm by isolating it from the helium bath. This was achieved by soldering the tube inside a 1.2 mm diameter copper tube which in turn was soldered to the hydrogen can; it was terminated about 15 mm above the vacuum case. The whole system was jacketed with a 4 mm diameter german-silver tube. When liquid helium was transferred to the Dewar vessel, the residual air inside the 4 mm tube solidified, leaving the copper tube vacuum insulated from the helium bath. Since the thermal conductivity of copper is very large compared with that of german silver, almost all of the capillary remained at 20 °K, the temperature of the hydrogen can. The volume of this capillary between the sample inlet valve and the calorimeter was 0.06 cm³.

The main pumping tube D , of 10 mm diameter german silver, was fitted with a radiation shield at its lower end, from which the calorimeter was supported by a nylon thread. Electrical leads of constantan wire ran inside tube D ; to minimize the heat leak to the calorimeter along them, they were tied to helium temperatures at the two copper cylinders S . These cylinders were soldered to the top of the vacuum case, and four short pieces of insulated copper wire were wound around each of them and secured with Araldite. The leads were then soldered to these copper wires, which in turn were connected to the calorimeter by short 36 s.w.g. constantan leads. The vacuum seal T was made with Wood's metal.

The calorimeter pot (see also figure 2) was connected to a pumping system, manometers, and helium and hydrogen supply cylinders through the 4 mm german-silver tube E . Just above the vacuum case a cooling coil Q , consisting of a single copper loop, was fitted. Inside the vacuum case, the tube diameter was reduced to 2 mm.

The evaporation rates of hydrogen and helium were 150 and 50 cm³/h of liquid, respectively. The running-out of helium was shown well in advance by the gas thermometer R , whose reading rose when the helium level fell below the top of the vacuum case.

(b) *The calorimeter*

The same calorimeter was used for both specific-heat and equation-of-state measurements. As shown in figure 2, it had two compartments, the sample space and the pot, the latter acting as a container for liquid hydrogen or helium used for cooling purposes and for thermometer calibrations. The calorimeter was assembled from four parts: the main body, the top, the pot drum and the thermometer drum. To ensure short equilibrium times, all the parts were made of copper.

The main body was in the form of a cylinder with an integral bottom. It was necessary to minimize dilation under pressure without incurring too large a heat capacity; a moderate wall thickness (3.3 mm) was therefore used, and reinforcing rings were left at both ends and also in the middle. The top was screwed into the main body and soft soldered. To form the pot, a drum was slid on to the main body and soldered to the bottom and middle reinforcing rings, thus providing an annular space of 12 cm³ volume. The heater and constantan thermometer were wound and bonded on to a separate drum which fitted over the upper part of the main body and was secured to one of the rings with Wood's metal. This form of

construction minimized the risk of straining the thermometer wire through dilation of the calorimeter under pressure. A further advantage was that the adhesive used for bonding the windings (Araldite 985 E) could be properly baked without risk of deterioration of the various soft-soldered joints of the assembly.

The constantan thermometer was wound from a reel of wire kept in the laboratory for this particular purpose; it has the merit of a linear resistance-temperature characteristic between 8 and 20 °K. An ordinary radio resistor was used as the carbon thermometer. The paint was scraped off, and an enamelled copper wire wound round the surface and secured with Araldite. Both ends of the wire were soldered to the top of the calorimeter to provide good thermal contact. The equilibrium time between the thermometers and the calorimeter was less than 1 sec.

To facilitate the attainment of thermal equilibrium within the helium specimen, the calorimeter was packed with thin copper wire which occupied about 30 % of the available space. The volume remaining for the specimen was deduced from measurements at room temperature by means of the apparatus of Berman & Poulter (1952) in which gas volumes are measured at constant pressure. The values were found to be reproducible to 0.1 %. The correction for the thermal contraction of the calorimeter at low temperatures was made from the data of Beenakker & Swenson (1955). This correction amounted to 0.98 % down to 20 °K, below which temperature the contraction is negligible. The final value adopted for the volume was 21.446 cm³.

(c) *Thermometer calibrations*

Before each experiment, the two thermometers were calibrated between 12 and 20 °K against the vapour pressure of normal hydrogen (75 % *ortho*hydrogen) condensed to the calorimeter pot. The vapour pressure relation used was that of Woolley, Scott & Brickwedde (1948). Calibrations against the vapour pressure of helium were carried out in a similar manner, using the temperature scale T_{55E} proposed by Clement, Logan & Gaffney (1955). For the purposes of this work, it is not necessary to distinguish between this scale and its contemporary T_{L55} (van Dijk & Durieux 1955) or the subsequent T_{58} (Brickwedde 1958).

The constantan thermometer was inferior to the carbon from the point of view of sensitivity, but showed better stability. For this reason, it had been intended to use it in the range from 4 to 10 °K where direct calibration was not possible during every experiment. Some preliminary tests showed, however, that the calibration curve of the carbon thermometer in this range could be reduced to the very simple form

$$(R - R_0)^{-1} = AT + B,$$

where R_0 , A and B are constants. Furthermore, the small changes of calibration which occurred between experiments affected the value of R_0 only, so that only one calibration point was needed in each experiment to make full allowance for the changes (Lounasmaa 1958). The carbon thermometer was therefore used at all temperatures up to 10 °K; at higher temperatures the carbon resistance became inconveniently small, and the constantan thermometer was preferred.

3. SPECIFIC HEAT MEASUREMENTS AND RESULTS

(a) Heat capacity measurements

The helium samples were taken from a cylinder of commercial helium gas, and were purified by passing them through two charcoal cleaners, one cooled with liquid oxygen and the other with liquid hydrogen. Before each experiment, the charcoal was re-activated by heating to 230 °C under vacuum for about 2 h.

With the whole apparatus cooled to near 20 °K, the sample was admitted and the exchange gas removed from the vacuum case. Liquid hydrogen was then condensed into the calorimeter pot, the necessary calibrations performed, and the remainder of the hydrogen evaporated, leaving the calorimeter at about 14 °K. Liquid helium was then transferred, and some condensed into the calorimeter pot. Evaporation of this under reduced pressure cooled the calorimeter below the bath temperature, and by a combination of pumping and heating a suitable starting temperature (2 to 3 °K) could be achieved and maintained until the evaporation was complete. The calorimeter then warmed up slowly owing to heat leaks from the main helium bath and the heat dissipation in the thermometer. Since exchange gas had only been used at temperatures where its removal was easy, a good vacuum was achieved and the heat leaks were small, corresponding to temperature drifts of a few millidegrees per minute at worst. For measurements above 12 °K, liquid hydrogen replaced helium in the inner Dewar vessel.

The temperature drift was first recorded, and the heating current then switched on for a period of 1 or 2 min and measured with a potentiometer. Next, the after drift was observed; this was also used as the fore drift for the second heat capacity point. The heating time was recorded with a stop-watch reading to 0.1 sec. To avoid errors in timing, the watch was operated by the heater current switch. The resistance of the heater was measured as a function of temperature in advance. Measurements were continued in this manner until the temperature range was covered; the experiment could then be repeated by again condensing liquid helium to the calorimeter pot and following the procedure just described. When the measurements were completed, the whole cryostat was warmed up with the calorimeter connected to a cylinder of accurately known volume immersed in a water-bath. The pressure rise in this cylinder determined the amount of sample and hence also its density.

Some difficulty was experienced with spontaneous oscillations of helium in the filling tube *B* causing large heat leaks to the calorimeter. They were suppressed by changing the tube geometry. A full account of this phenomenon has been given by Lounasmaa (1959*a*).

(b) Calculation of heat capacities

Heat capacities were calculated from the equation

$$C = i^2rt(d\eta/dT)/\Delta\eta,$$

where *i* is the heating current, *r* the heater resistance, and *t* the heating time. The thermometric parameter η is just the thermometer resistance when the constantan thermometer is used, and is $1/(R - R_0)$ for the carbon. The evaluation of $\Delta\eta$, the change of thermometric parameter due to the measured quantity of heat, was made by the simple method appropriate to the case of small heat leaks: the fore and after drifts were extrapolated to the middle of the

heating period, and $\Delta\eta$ taken as the difference between the two extrapolated values of η . The heat capacity was then referred to the mean temperature of the heating interval.

(c) *The expansion correction*

Measurements of the true C_V naturally require a vessel which does not expand with increase of pressure or temperature, a condition which can never be satisfied completely. In the present temperature range, thermal expansion could be neglected, but a small correction was required for dilation under pressure according to the relation

$$C_V = \frac{1}{n} \left\{ C_n - T \left(\frac{\partial p}{\partial T} \right)_V^2 \frac{dV}{dp} \right\}.$$

Here C_n denotes the measured heat capacity of n moles of helium, $(\partial p/\partial T)_V$ is the state property of helium whose measurement is described in § 4, and dV/dp is a property of the calorimeter.

A separate experiment was undertaken to measure dV/dp at room temperature, and a value of $7.8 \times 10^{-4} \text{ cm}^3 \text{ atm}^{-1}$ found for it. As the elastic constants of copper change by only about 10 % down to helium temperatures (Allen 1952), and the correction was always less than 0.5 %, this measured value could be used at all temperatures. When the density of the sample was computed, the change in the calorimeter volume with pressure was taken into account by assigning a mean pressure to each run.

(d) *Results of C_V measurements*

All the measured specific heat points are listed in table 1,* and most of them have been used in the construction of figure 3. The form of the curves is quite simple except at the low-temperature end, where three types of behaviour can be distinguished: for the three highest densities, the curves terminate at the liquid–solid equilibrium line; for the three following densities, the high-temperature tail of the λ -anomaly is seen; for low densities, the curves cross the liquid–vapour line. It can also be seen that over much of the range the specific heats are very different from that of an ideal monatomic gas ($12.47 \text{ J deg}^{-1} \text{ mole}^{-1}$). At higher temperatures, the curves converge towards this value, but cross at about 17°K , in agreement with the observations of Dugdale & Simon (1953) near the melting curve.

To prepare a table of C_V for convenient values of density and temperature, smoothed values have to be used. Little is known about the form of the relationship, so careful graphical treatment of the experimental data was considered to be the best method of obtaining the final smoothed values. Experimental points belonging to each run and hence to a common density were first joined by a smooth curve as in figure 3. Smoothed values were then obtained by reading C_V from these curves at fixed temperatures. Next, to connect the different experiments with each other, these values were plotted as graphs of C_V against density and points belonging to the same temperature joined by a smooth curve. Some examples of this are given in figure 4. Values of C_V at fixed temperatures and densities were then taken from these curves. Finally, a new plot of C_V against temperature was made. Only very small deviations from smoothness were observed at this stage, and the values thus obtained were listed as a table. Differences were calculated and small rounding-off errors still present in the last decimal corrected. The final values of C_V are listed in table 2.

* The tables are at the end of the paper.

A few entries in this and some subsequent tables are given in parentheses; these entries correspond to states where the pressure exceeds 100 atm. No measurements were made under these conditions, and the values given were obtained by extrapolation. Figure 4 shows that the specific heat is very nearly a linear function of density at high temperatures and densities, so that short extrapolations could be made with reasonable confidence.

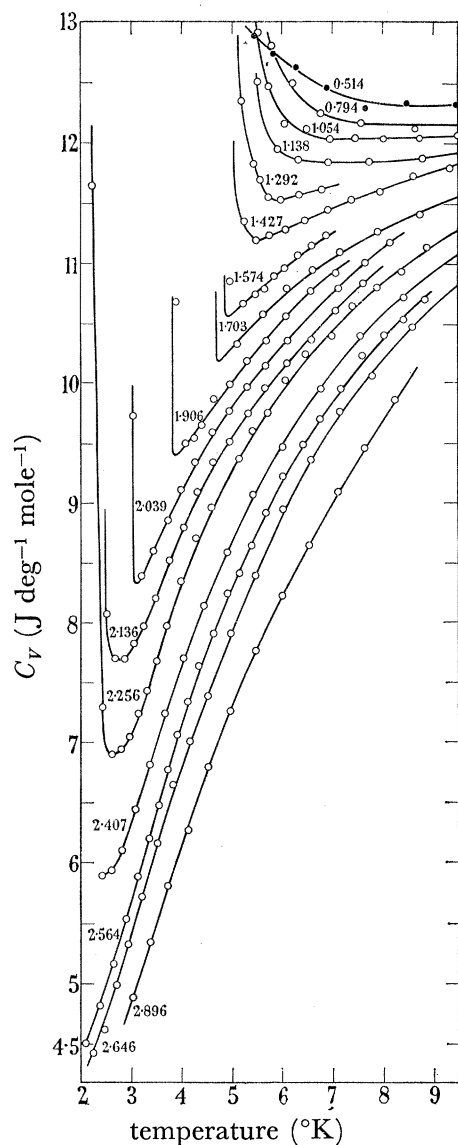


FIGURE 3. Experimental values for C_V .
The number on each curve is the reduced density.

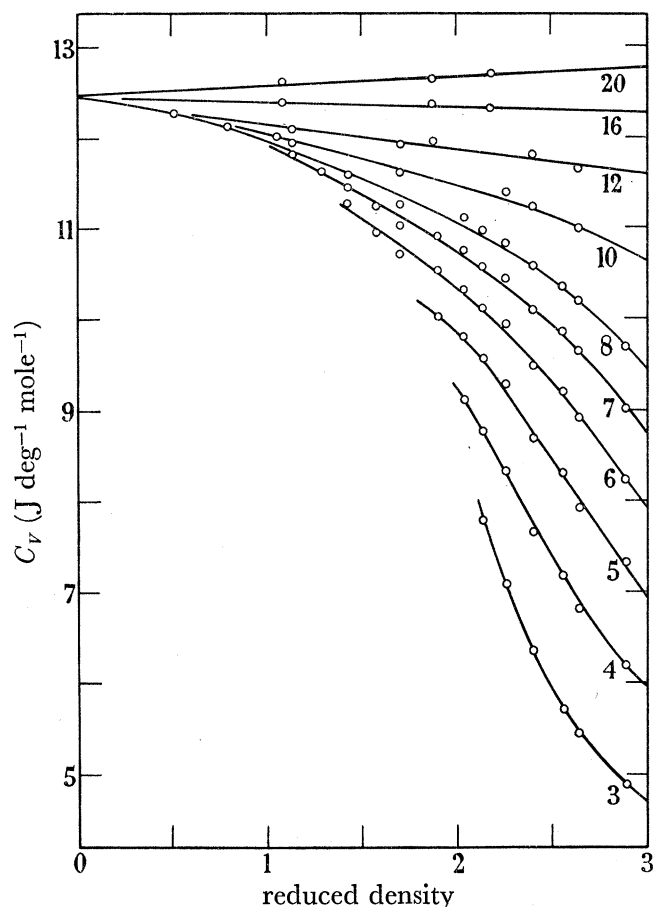


FIGURE 4. C_V as a function of density. Each curve is marked with the temperature in °K.

Earlier measurements of C_V have been made by Keesom & Clusius (1932) and by Keesom & Keesom (1935, 1936*a*) at temperatures below 4.3 °K and for densities close to 2.4 and 3.0 in our units. When recalculated to the T_{55E} temperature scale, these data show differences of $\pm 5\%$ from the present ones. Measurements by Eucken (1916) at temperatures between 18 and 30 °K and for densities between 0.5 and 2.0 lie within -1.4 to $+3.0\%$ of the present data in the short range of overlap, and are particularly useful in showing clearly that C_V

increases with density at higher temperatures. No data exist for comparison in the 4 to 18 °K temperature range.

(e) *Accuracy of C_V measurements*

A rough idea of the accuracy of the experiments can be obtained from figures 3 and 4, where the scatter does not normally exceed 0.5 %. If Δ is the difference between an actual measured point and the corresponding smoothed value, then m , the root mean square value of Δ , is 0.77 %. The corresponding probable error of $\frac{2}{3}m$ is thus 0.5 %. In order to account for the scatter and to estimate possible systematic errors, the equation of § 3 (b), used to calculate heat capacities, is now analyzed.

The main uncertainty is probably in the thermometer sensitivity $d\eta/dT$; experience shows that an accuracy of 0.5 % can be claimed for this. The best possible accuracy in $\Delta\eta$ is obtained if temperature drifts are linear and small in comparison with the heating rate, and if equilibrium is established quickly after heating. The first requirement was always satisfied because of the large heat capacity of the sample and the good thermal insulation. The calorimeter temperature reached its equilibrium value in about 5 sec after heating. Thus both conditions were well fulfilled, and an accuracy of 0.3 % is claimed in $\Delta\eta$. The heater resistance r was about 440 Ω and was measured to 0.1 Ω ; the figure included half of the resistance of the leads between the heater and the blocks S (figure 1), on the assumption that half of the heat generated in them flows to the calorimeter. The accuracy of the effective value of r is estimated as 0.1 %, and similar values apply for both i and t . The potentiometer and its equipment were checked regularly, and the watch compared with the crystal controlled clock of the laboratory.

When C_V was calculated, additional sources of error arose from the expansion correction, the determination of the heat capacity of the empty calorimeter $C_{\text{cal.}}$, and the number of moles of sample n . The expansion correction was always so small that it could hardly affect the accuracy of C_V . $C_{\text{cal.}}$ was measured with the same precision as other heat capacities, and its effect on the final accuracy depends strongly on temperature. At helium temperatures $C_{\text{cal.}}$ was only 1 or 2 % of the total and therefore had no effect on the accuracy of C_V . Near 20 °K, however, $C_{\text{cal.}}$ was sometimes as much as 20 % of the total heat capacity, and in this case its maximum effect on the final accuracy is 0.5 %. The number of moles n was measured to 0.2 %. Combining the above estimates, the maximum errors in C_V become 1.2 % at helium temperatures and 1.7 % near 20 °K. The use of the measured mean specific heat in the temperature interval $T_2 - T_1$ as the true specific heat at the temperature $\frac{1}{2}(T_1 + T_2)$ can easily be justified as C_V changes relatively slowly with temperature.

The above considerations show that the probable error in the C_V values over the whole range covered in these experiments is about 1 % and somewhat less at the lower temperatures. As many of the sources of error were systematic, the relative accuracy is greater, probably about 0.6 %.

4. MEASUREMENTS ON THE EQUATION OF STATE

(a) *Additional apparatus and experimental procedure*

The measurements on the equation of state were carried out in the same cryostat and calorimeter as the C_V experiments. The object of these experiments was to obtain both a complete set of p - V - T data and directly measured values of the derivative $(\partial p/\partial T)_V$ throughout the chosen range, and the pressure measuring apparatus was designed accordingly.

Figure 5 is a drawing of this additional apparatus. V_1 and V_2 are two ballast volumes, about 200 cm³ each, made of heavy phosphor-bronze tube and immersed in a water-bath. M_1 and M_2 are two mercury in glass manometers 170 and 25 cm long, respectively. A_1, \dots, A_6 are various high-pressure needle valves and B_1 and B_2 special valves having negligible dead volume on the calorimeter (C) side. t.g. is a 10 in. dial standard test gauge and d.w.t. is a dead-weight tester. The gauge G was used only when filling the calorimeter to the required pressure. The tubing between the calorimeter, the sample inlet valve B_1 , the manometer M_2 , and the valve A_5 was made of german-silver capillary of 0.3 mm inside diameter. All other connexions were made of ordinary high-pressure copper tubing.

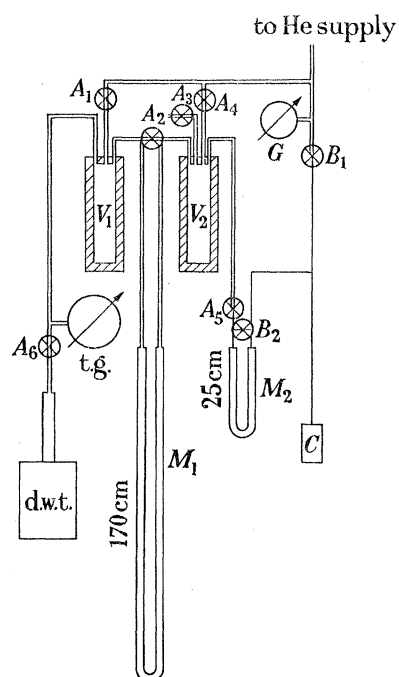


FIGURE 5. The apparatus for pressure measurements.

As the manometers had to withstand pressures up to 100 atm, their construction presented some difficulty. When a glass tube of 8 mm outside and 0.8 mm inside diameter was tested, breakage occurred at 350 atm. This, however, gives only a rough idea of the size of tube required because glass acquires internal strains, and after several applications of pressure the breaking stress may be only a small fraction of the initial value. The tested size was adopted more in hope than in certainty that it would be adequately strong, but only one breakage was experienced.

The experimental procedure for determining p and $(\partial p/\partial T)_V$ is fairly obvious from figure 5, and need only be explained briefly. For measurements of $(\partial p/\partial T)_V$ the pressure change produced in the calorimeter by a small, known, temperature change was measured with the differential manometers. During these measurements, the smaller manometer M_2 was used as a null indicator so that the volume on the calorimeter side remained constant. Pressure changes in the calorimeter were compensated by letting more gas into V_2 and measured by the resulting deflexion of M_1 . In this way, a number of points could be taken until the mercury in M_1 reached the top of the left-hand tube. The pressure in V_1 was then increased so as to deflect M_1 fully in the opposite sense, when the apparatus was

ready for further measurements. The pressure in V_1 was measured with the dead-weight tester below 45 atm and with the test gauge above this value.

For convenience, the experiments were done in two parts: first, measurements were made between 3 and 11 °K using the carbon thermometer and with liquid helium in the inner Dewar vessel; and secondly, between 10 and 20 °K, using the constantan thermometer and liquid hydrogen. The equilibrium time of M_2 after heating was only a few seconds. This was because of the good heat conductivity of the calorimeter and the smallness of the amount of gas which had to be transported through the 0.3 mm capillary.

(b) *Calculation of results*

During each experimental run the density of the sample remained constant (except for a small dilation of the calorimeter) and the relation between pressure and temperature was studied. The mass of the sample was found in the same way as for the specific heat measurements. The dead volume outside the calorimeter during the measurements included part of the smaller manometer M_2 and the tubing as far as the valves B_1 and B_2 , representing in all a volume of 0.3 cm³. As almost all of this was at room temperature, the amount of sample outside the calorimeter was only about 0.2 %, depending on the calorimeter temperature. A correction was applied for this.

To all pressures read from the dead-weight tester and the gauge, atmospheric pressure was added. Absolute pressures in the calorimeter differed from this only by the amount indicated by the manometer M_1 . Changes of pressure were measured with the valve A_6 closed, and were therefore given by the change of reading of M_1 , corrected for the compression of the gas in V_1 and the left-hand manometer tube. In order to calculate this correction, it was necessary to determine the volume of V_1 and the associated tubing. Thereafter, the correction depends only on the equation of state of helium at room temperature, and although it is quite large at the higher pressures, it could always be made to the required accuracy.

Since the isochores are not strongly curved, it is permissible to write $(\partial p/\partial T)_{\text{meas.}} = \Delta p/\Delta T$. Here Δp and ΔT are the measured pressure and temperature changes; the magnitude of the latter was usually about 0.2 °K at the lower temperatures, increasing to 1 °K at the higher temperatures. As previously explained, the calorimeter expanded slightly under pressure; the true $(\partial p/\partial T)_V$ is given by

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial p}{\partial T}\right)_{\text{meas.}} \left[1 - \frac{1}{n} \left(\frac{\partial p}{\partial V}\right)_T \frac{dV}{d\rho}\right].$$

The values of $(\partial p/\partial V)_T$ could be obtained from uncorrected measurements at different densities since the correction does not exceed 2% and is normally considerably smaller. The expansion of the volume V_1 under pressure, the compressibility of mercury, the expansion of the glass capillary of the manometer, and the effect of temperature variations in the water-bath around V_1 were all found to be negligible.

(c) *Results of isochore measurements*

The results of pressure–temperature measurements along isochores are given in table 3 and many of the points are also plotted in figure 6. The isochores are concave upward at first, but approach to straight lines at higher temperatures. The data were smoothed graphically in

the same way as for the specific heat. Figure 7 shows the pressure–density plot, and the final smoothed values are given in table 4. For pressures less than 25 atm, the pressure is given to two decimal places, but for higher pressures the second decimal is given only as 0 or 5.

When this table was being compiled, some difficulty was encountered because of the limited accuracy of the dead-weight tester at the lowest pressures. However, when the isochores were extrapolated to the liquid–vapour line the present results seemed to confirm the densities given by Clement *et al.* (1955) in the table of T_{55E} . These values are based on the

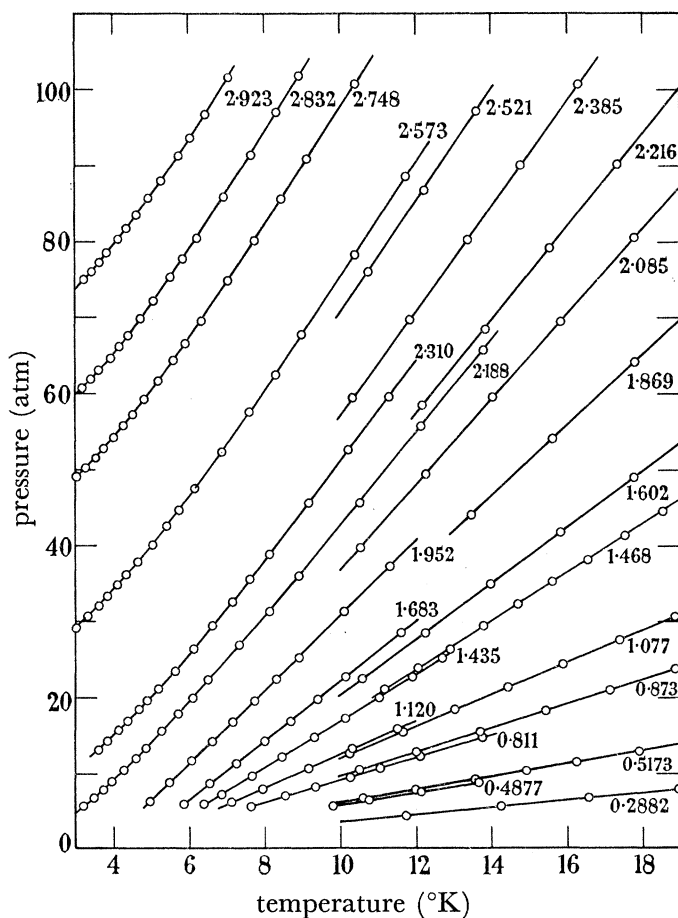


FIGURE 6. Experimental pressure values. Each curve is marked with the reduced density.

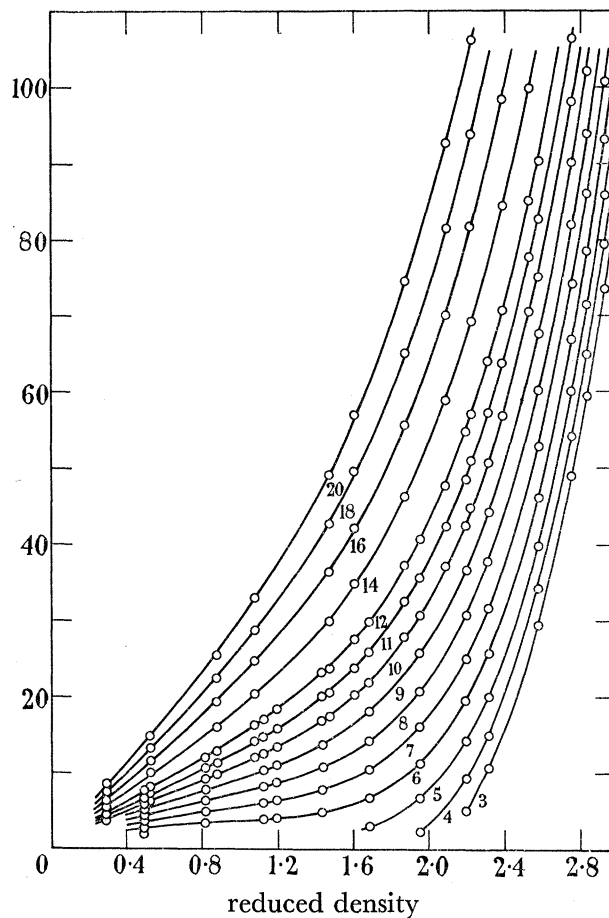


FIGURE 7. The pressure as a function of density. The number under each curve is the temperature in °K.

densities in Keesom (1942, p. 205) but are referred to the new temperature scale; they were taken to be correct and used to fix the low-temperature end of those isochores which meet this phase boundary. The difference between the Clement densities and those of Kerr (1957) is 0.2%; the present measurements are not accurate enough at low pressures to distinguish between them.

When the density approaches zero, the behaviour of the sample must tend to that of an ideal gas $p = 1.410\rho T$. This equation was applied to fix the low-pressure end of the p - V - T table in the limit of zero density, where

$$p = 0, \quad (\partial p / \partial T)_V = 0, \quad (\partial p / \partial \rho)_T = 1.410 T.$$

The present p - V - T data have been fitted to the Benedict-Webb-Rubin equation of state (Lounasmaa 1959*b*). It was found that this equation represents the state properties of helium rather well.

(*d*) *Earlier measurements on the equation of state*

A survey of the earlier measurements for establishing the equation of state of helium is given by Keesom (1942, pp. 28-47). More recently, isotherms at low pressures have been measured in the liquid-helium temperature range in order to determine the second virial coefficient of the gas (Keller 1955). Of the earlier experiments, there are two which can be compared extensively with the present work. Keesom & Keesom (1933*b*) measured isochores between 1.15 and 4.2 °K for pressures up to about 35 atm. The present densities are

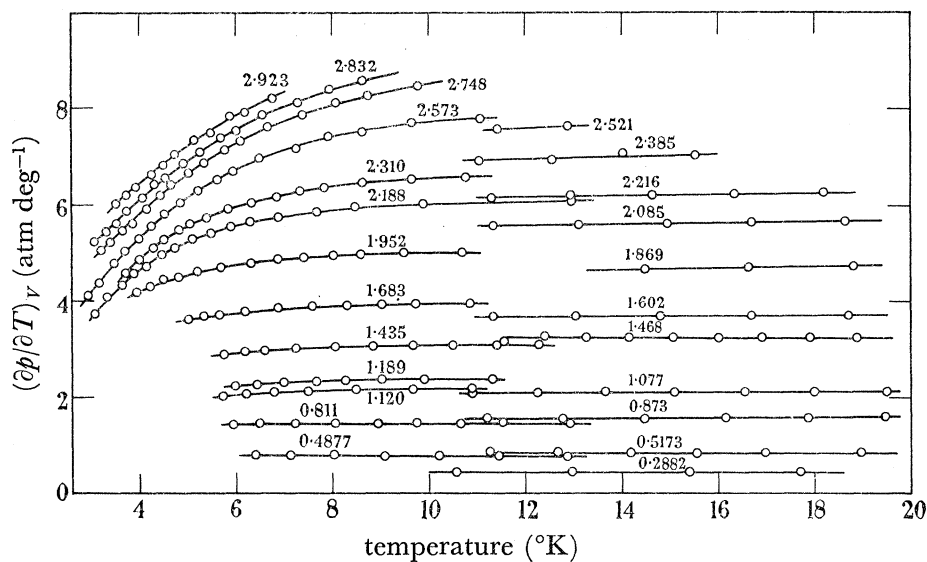


FIGURE 8. Measured values of $(\partial p/\partial T)_V$. The number on each curve is the reduced density.

about 0.3% higher, the differences becoming smaller at low densities. Holborn & Otto (1926) measured isotherms at 15.16 and at 20.36 °K; their density values are generally a little higher than the present ones, the mean value of the differences being 0.35%. Densities read from the entropy diagrams of Zelmanov (1944*a*) and of Keesom *et al.* (1953) can also be compared with the present results. The agreement at 20 °K is good because the densities of Holborn & Otto were adopted in both cases. In the intermediate region, where both of these diagrams rely on interpolation, quite large discrepancies are found. In particular, the values given by Zelmanov appear to be too low. This is perhaps not surprising, as the method used by him to calculate the volume was rather indirect.

(*e*) *Results of $(\partial p/\partial T)_V$ measurements*

Measured values of $(\partial p/\partial T)_V$ are listed in table 5 and most of them are plotted in figure 8. This derivative is strongly dependent on temperature below 5 °K, but becomes remarkably independent of it above 10 °K. This latter behaviour is reminiscent of an ideal gas, though the numerical values are considerably different. The results were smoothed graphically in much the same way as already explained for C_V and p and smoothed values are given in table 6. Owing to the good internal consistency of the experimental results and the

relatively simple form of the variation with density and temperature, this presented no particular difficulty.

The measurements of p and those of $(\partial p/\partial T)_V$ were partly independent, depending chiefly on the dead-weight tester and the mercury manometer, respectively. For intercomparison, pressure changes over wide temperature intervals were taken from the p - V - T data and compared with the values obtained by integrating the corresponding $(\partial p/\partial T)_V$ curves. No disagreement exceeding 0.5% was found. The measurements of $(\partial p/\partial T)_V$ and C_V could in principle be compared through the relation $\sigma C_V/\partial V = T(\partial^2 p/\partial T^2)_V$, but the accuracy with which the derivatives can be calculated is too low for the comparison to be instructive. Comparison can also be made with a diagram of $(\partial p/\partial T)_V$ obtained by Keesom & Keesom (1933*c*) by differentiating along measured isochores. The scale of this diagram is very small, but it seems to give considerably smaller changes of $(\partial p/\partial T)_V$ with density than the present data.

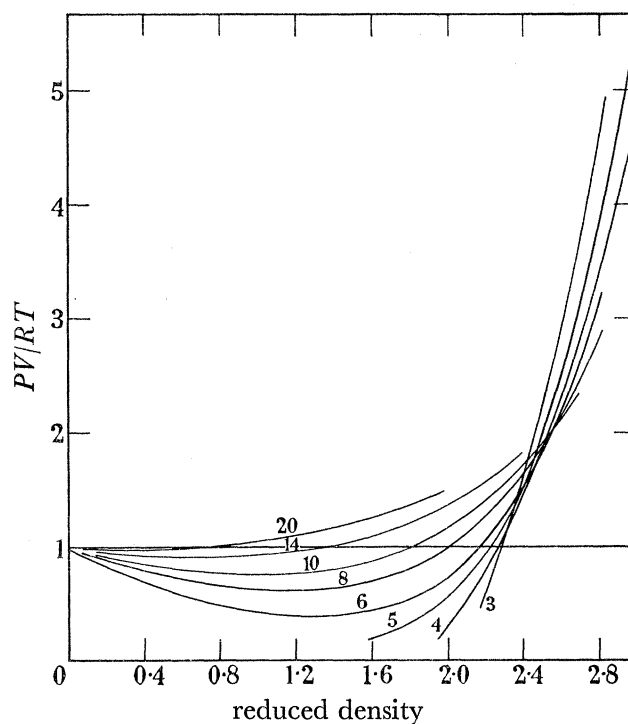


FIGURE 9. The compressibility factor. The number on each curve is the temperature in °K.

(f) *Derived quantities*

Besides $(\partial p/\partial T)_V$, the other state derivatives $(\partial V/\partial p)_T$ and $(\partial V/\partial T)_p$ are of great utility in thermodynamic calculations, and can be derived from the present data. Only $(\partial V/\partial p)_T$ is considered here, but $(\partial V/\partial T)_p$ can be found immediately from the relation

$$(\partial V/\partial T)_p = -(\partial p/\partial T)_V (\partial V/\partial p)_T.$$

For the evaluation of $(\partial V/\partial p)_T$, $(\partial p/\partial \rho)_T$ was first calculated from the data of table 4; second differences were taken into account by a modified Newton-Gregory formula. Values of $(\partial V/\partial p)_T$ were then obtained from the relation

$$\left(\frac{\partial V}{\partial p}\right)_T = \frac{dV/d\rho}{(\partial p/\partial \rho)_T} = -\frac{58.18}{\rho^2(\partial p/\partial \rho)_T}$$

and are given in table 7.

The compressibility factor pV/RT can be found by using values from table 4. The results are given in table 8 and are shown in figure 9.

(g) *Accuracy*

The accuracy of the p - V - T measurements is not very easy to estimate. A direct measure of the consistency of the data and the success of the smoothing can be obtained by computing the root-mean-square deviation of the measured points from the corresponding smoothed values, which yields $m = 0.93\%$ and the probable error $\frac{2}{3}m = 0.6\%$. This, of course, takes no account of systematic errors, and it is better to consider each experimental uncertainty separately.

The accuracy of pressure measurement up to 45 atm depended only on the dead-weight tester. This instrument was stated by the manufacturers (Barnet Instruments Ltd) to be accurate to 0.1%. Measurements were found to be reproducible to this figure for pressures greater than about 4 atm, but a somewhat lower value obtained for smaller pressures. Here the accuracy could readily be checked by using the instrument to measure the saturation vapour pressure of the liquid, and it was found that an accuracy of 0.01 atm was achieved. For pressures above 45 atm, the instrument became too cumbersome to use continuously, so it was used to calibrate the pressure gauge which replaced it. This gauge could be read with an accuracy of 0.1 atm, or about 0.2% near 50 atm pressure. From these considerations, the accuracy of pressure measurements is estimated to be 0.2% at all pressures above 4 atm.

To calculate the molar volume V or the density ρ , it was necessary to know the calorimeter volume under experimental conditions and the number of moles of sample. The former was known with an accuracy of 0.1% (see § 2*b*) and the latter was measured to 0.2%. Consequently, the maximum error in V or ρ was 0.3%, an error which still corresponds to a rather pronounced uncertainty in the pressure at the higher densities. The accuracy of temperature measurement was 0.01 °K above 4 °K, and about 0.003 °K below. An error of 0.01 °K in temperature corresponds to an uncertainty of 0.1 to 0.2% in the pressure. Thus an overall accuracy of 1% can be claimed in the values of pressure corresponding to exact values of density and temperature. The relative accuracy should be somewhat better, about 0.6% in accordance with the calculated probable error.

When estimating the accuracy of the $(\partial p/\partial T)_V$ measurements, it is too complicated to analyze in detail the formulae used in the computations. The root-mean-square deviation m was again calculated by comparing the measured points with the smoothed data. The result was $m = 0.68\%$, giving a probable error of about 0.5%, which indicates good internal consistency in the data. Since $(\partial p/\partial T)_V$ changes relatively slowly with temperature, errors in temperature measurement should have little effect on the values of the derivative, but uncertainties in density have slightly bigger effects. It is concluded that the accuracy of these measurements is somewhat better than that of the p - V - T data. Allowing for errors in the quantities entering the equations used to calculate $(\partial p/\partial T)_V$, 0.7% is a fair estimate for the maximum error in table 6. At densities below $\rho = 0.8$, the accuracy is poorer, mainly because there were rather few experimental runs in this region.

The other derivative $(\partial V/\partial p)_T$, is considerably less accurate, as is evident from the method of calculation. The errors in pV/RT are just those of p at fixed V and T which have just been discussed.

5. THERMODYNAMIC FUNCTIONS

(a) The entropy diagram

The measurements of C_V and $(\partial p/\partial T)_V$ can now be combined to construct the entropy diagram of helium in the single-phase region between 2.5 and 20 °K for pressures up to 100 atm. As a starting point, the entropy of the liquid in equilibrium with its vapour at 4.275 °K was taken to be 14.18 J deg⁻¹ mole⁻¹ (Hill & Lounasmaa 1957). This particular temperature was chosen as being the one closest to the normal boiling point of 4.214 °K and corresponding to a convenient value of the density ($\rho = 1.80$). The measurements were then used in the following way:

- (i) the entropy for $\rho = 1.80$ and $T = 6$ °K was calculated from the C_V data,

$$\Delta S = \int C_V/T \, dT;$$

- (ii) the entropy for $T = 6$ °K and all other densities was found from $(\partial p/\partial T)_V$ data,

$$\Delta S = \int (\partial p/\partial T)_V \, dV;$$

- (iii) entropy values at all other temperatures were calculated from the specific heats.

The results of these calculations are given in table 9.

The choice of $\rho = 1.80$ and $T = 6$ °K as the basic lines for calculations appears quite arbitrary, but there are several factors which make it an advantageous one. The C_V values along the line $\rho = 1.80$ change quite slowly with temperature up to 6 °K which assures a reliable result for the integral. The temperature at which the entropies along different isochores are interrelated should be high enough to be above the disturbing effects of the critical point, yet still as low as possible so as to avoid loss of accuracy in the low-temperature values due to massive subtraction. Taking both requirements into account, the choice of 6 °K appeared to be the most favourable.

The entropy diagram is shown in figure 10; it includes lines of constant density, pressure, and enthalpy. In this diagram, the integral $\int T \, dS$ can easily be evaluated along these lines, and gives the quantity of heat transferred in the corresponding process. This diagram is shown chiefly to give a clear picture of the dependence of the entropy on the more important parameters; when numerical values are required they should preferably be taken from the table. The extrapolated region (§ 3d) is not included in the diagram.

(b) Discussion of the entropy diagram

The entropy diagram can be checked in various ways. As a check on the internal consistency, entropy values can be calculated by different routes. If the values are first found for $\rho = 1.80$ and all temperatures from C_V data, and then for all other densities from $(\partial p/\partial T)_V$ data, discrepancies are found which vary from about 0.2 % for large values up to 1 % for the smallest. The reason for the increase is that in the latter case two comparable quantities have been subtracted to yield the final, small value. A second type of comparison can be made at the liquid–vapour and the liquid–solid equilibrium lines. The entropy of the liquid at densities $\rho = 1.4, 1.6$ and 2.0 can be obtained from the measurements of Hill &

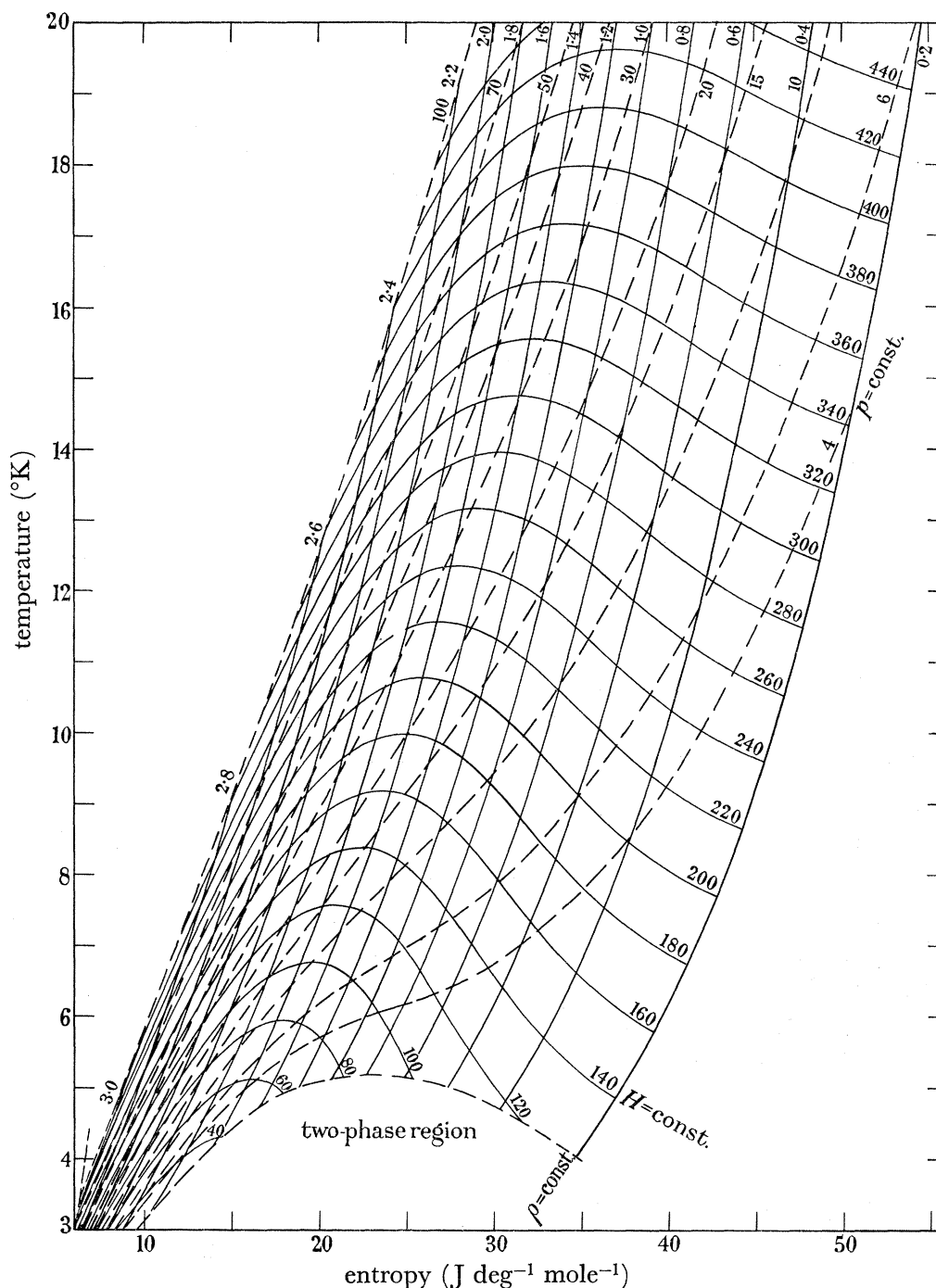


FIGURE 10. The entropy diagram of helium. Lines of constant density ($\rho = 0.2$ to 3.0), constant pressure ($p = 4$ to 100 atm), and constant enthalpy ($H - H_0 = 40$ to 440 J mole^{-1}) are shown.

Lounasmaa (1957); for densities $\rho = 0.2$, 0.4 and 0.6 , the same measurements combined with the latent heat values of Berman & Mate (1958) can be used. The entropy of the fluid in equilibrium with the solid can be calculated for $\rho = 2.8$ and 3.0 by combining the measurements of Keesom & Keesom (1936*b*) and Swenson (1952). The discrepancies found are of the same order of magnitude as those just mentioned. Finally, the entropies at the lowest densities can be compared with those of an ideal gas. The differences so found are of the order of a few parts per cent, and increase with density. Extrapolation of the difference to zero

density suggests agreement to a few tenths of 1 %, though this is probably the least accurate check because of the lower accuracy of the $(\partial p/\partial T)_V$ measurements at low pressures.

The accuracy of the entropy values is the same as that of the quantities C_V and $(\partial p/\partial T)_V$ on which it depends. Their precision was estimated (§§ 3*e* and 4*g*) to be 1.0 and 0.7 %, respectively. The entropies should therefore not be in error by more than 1 %, except, perhaps, at the two lowest densities tabulated. The internal accuracy of the values is somewhat higher, about 0.7 % when judged from the probable error of the C_V and $(\partial p/\partial T)_V$ measurements.

The entropy values given by Zelmanov (1944*a*) tend to be too low in the liquid-helium region and too high at higher temperatures when compared with the present measurements. The differences are sometimes as much as 4 to 6 %. Much better agreement is found with the entropy diagram of Keesom *et al.* (1953). The differences do not exceed 2 % and are normally less in the range up to $\rho = 1.8$ which that diagram covers.

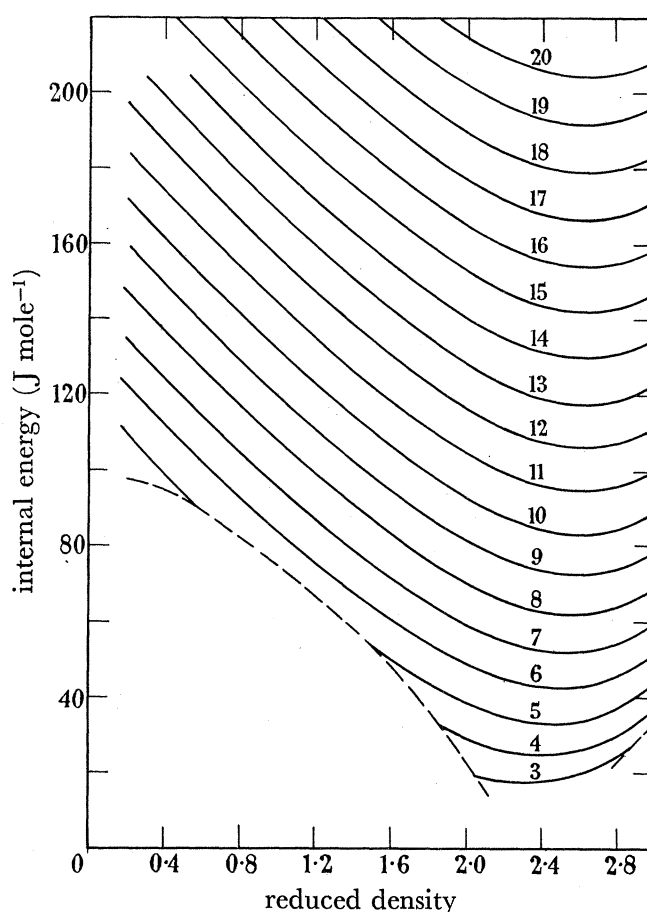


FIGURE 11. The internal energy $U - U_0$. The number on each curve is the temperature in °K; the dashed curves show the boundaries of the two-phase regions.

(c) *The internal energy*

Measurements of the specific heat of liquid helium under saturation vapour pressure (C_S) by Kramers, Wasscher & Gorter (1952) and by Hill & Lounasmaa (1957) were used to evaluate the internal energy U of the liquid according to the relation

$$U - U_0 = \int_0^T C_S dT - \int_{V_0}^V p dV.$$

The second term on the right, in which p is the vapour pressure and V the molar volume of the liquid, is only a small correction, its value being less than 1 % of the term in C_s . U_0 is the zero point energy of the liquid under zero pressure. When this equation was integrated between 0 and 4.275 °K, the result obtained was 36.90 J mole⁻¹, and this value was used as the starting point for calculations.

Internal energies were calculated in much the same way as entropies. For integration along the isochore $\rho = 1.80$ between 4.275 and 6 °K, the relation

$$\Delta U = \int C_v dT$$

was used; then for the constant temperature integration at 6 °K

$$\Delta U = T \int (\partial p / \partial T)_v dV - \int p dV$$

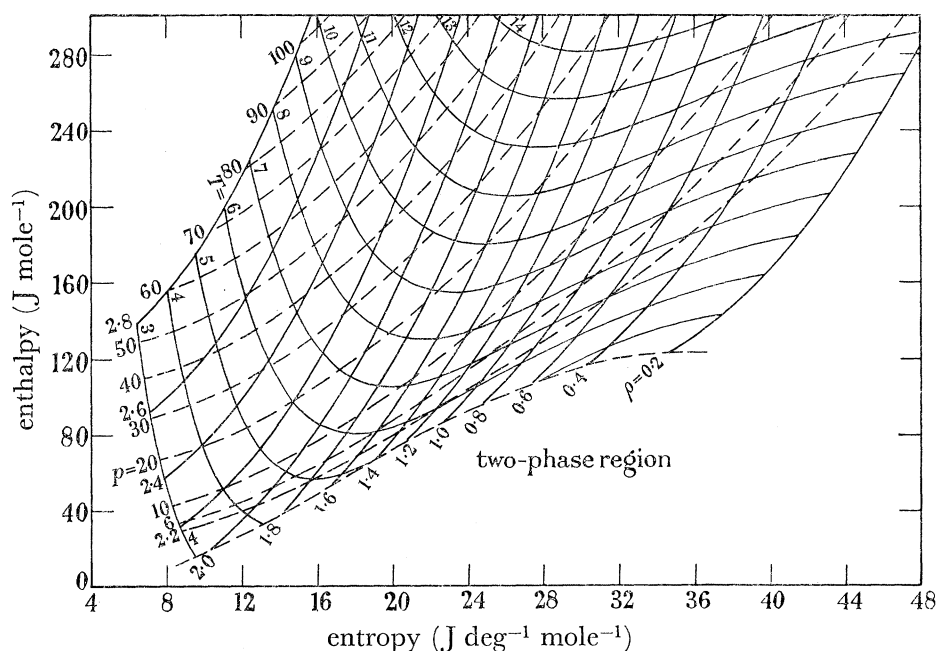


FIGURE 12. The Mollier diagram of helium. Full curves are isotherms from $T = 3$ to 14 °K and isochores from $\rho = 0.2$ to 2.8, dashed curves are isobars from 4 to 100 atm.

was employed; and finally, the diagram was completed by further integration along isochores. The results are given in table 10 and are shown in figure 11 in the form of curves of internal energy against density at various temperatures. These curves show minima at points where $(\partial p / \partial T)_v = p/T$.

The internal energy values were checked in much the same way as those of the entropy, and similarly good agreement was found.

(d) *The enthalpy*

The enthalpy H was calculated from the equation

$$H - H_0 = U - U_0 + pV,$$

with the values of $U - U_0$ taken from table 10 and of pV from table 8. The results are given in table 11, where in accordance with normal practice the values are given in terms of pressure

and temperature. The value of the enthalpy at the normal boiling point is $39.04 \text{ J mole}^{-1}$, this being 0.6% lower than the value given by Keesom (1942). The same author also quotes values of the enthalpy at 14 and 15 °K for pressures at which helium liquefiers are operated; these values are about 1% higher than the present ones. The only other comparison available is by extrapolation to the ideal gas state, where the differences are found to tend satisfactorily towards zero. Figure 12 shows the Mollier (entropy–enthalpy) diagram.

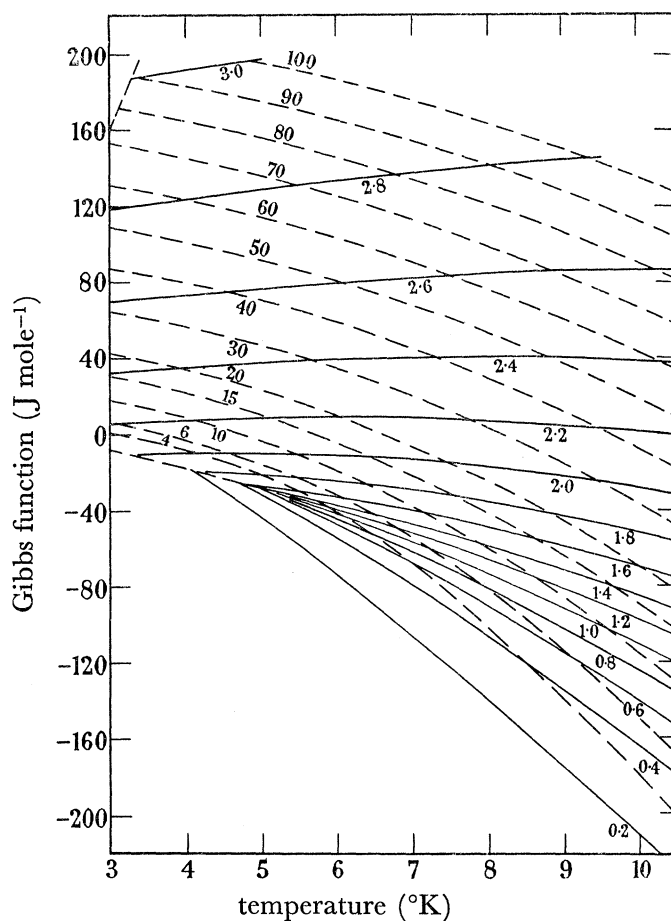


FIGURE 13. The Gibbs function $G - G_0$. Full curves are isochores labelled with the reduced density, dashed curves are isobars labelled with the pressure in atmospheres.

(e) *The Joule–Thomson inversion curve*

The inversion temperature is defined by the equation $(\partial T/\partial p)_H = 0$, the derivative being the Joule–Thomson coefficient. According to thermodynamics, points on the inversion curve satisfy the equation $(\partial V/\partial T)_p = V/T$. By using values of $(\partial V/\partial T)_p$ (see § 4f) the equation of the inversion curve was calculated and found to be

$$p = -21.0 + 5.436 T - 0.1324 T^2$$

when p is in atmospheres. Baehr (1956) has recently calculated inversion pressures from Zelmanov's (1940) results; a comparison, together with the values from Keesom (1942, p. 248) is given below. According to our equation, the maximum inversion pressure is 34.8 atm at $T = 20.5 \text{ °K}$.

THE JOULE-THOMSON INVERSION PRESSURE (ATM)

temperature (°K) ...	8	10	12	14	16	18	20
present results	14.0	20.1	25.2	29.2	32.1	33.9	34.8
Bachr	15.8	21.2	25.5	30.0	35.0	38.4	36.8
Keesom	—	—	26.5	29.7 ₅	32.7 ₅	34.7 ₅	—

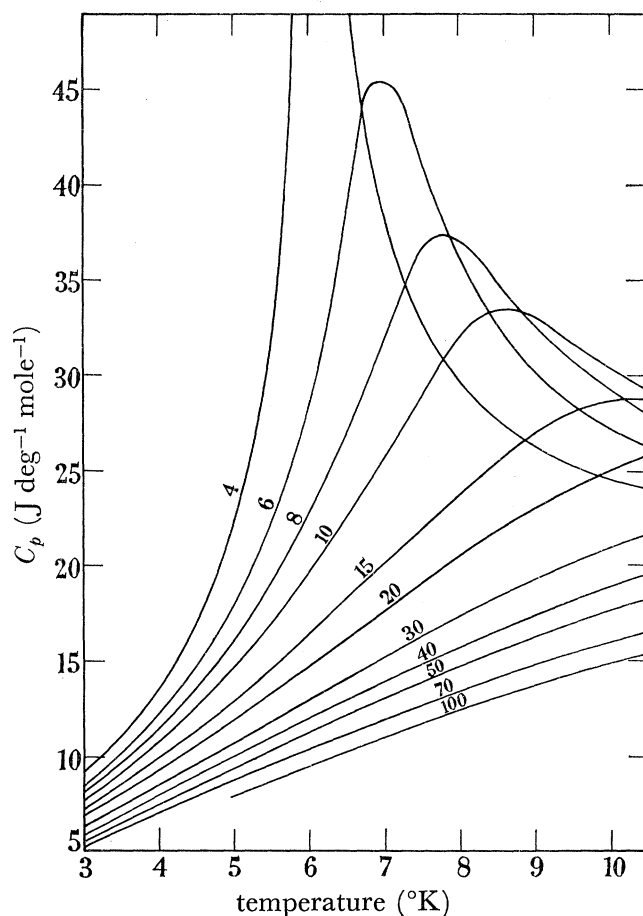


FIGURE 14. The specific heat at constant pressure. Pressure values in atmospheres are marked on the curves.

(f) *The free energy and Gibbs function*

The free energy F was calculated from the equation $F - F_0 = U - U_0 - TS$, with values of $U - U_0$ from table 10 and of TS from the entropies of table 9; the results are given in table 12.

The Gibbs function G can easily be found from either $G - G_0 = H - H_0 - TS$ or $G - G_0 = F - F_0 + pV$. This function is not tabulated, but some values are shown in figure 13. In this figure the two-phase region degenerates into a single line, since the Gibbs function is the same for the two phases.

At the normal boiling point, the following values were obtained: $F - F_0 = -22.78 \text{ J mole}^{-1}$, $G - G_0 = -19.53 \text{ J mole}^{-1}$.

(g) *The specific heat at constant pressure*

The specific heat at constant pressure C_p was calculated from the relation

$$C_p = C_v - T(\partial p / \partial T)_V^2 (\partial V / \partial p)_T$$

and the data of tables 2, 6 and 7. The results are given in table 13 and are plotted in figure 14.

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Near the critical point $(\partial V/\partial p)_T$ assumes high negative values, while C_V and $(\partial p/\partial T)_V$ change only slowly. Thus pronounced maxima in C_p are found in this range; these become lower and shift to higher temperatures with increase of pressure. In fact, if the co-ordinates of the maxima are plotted on a pressure-temperature graph, the helium vapour-pressure curve can reasonably be extrapolated through these points. Similar results have been obtained by Jones & Walker (1956) for argon.

The accuracy of C_p is inevitably much less than that of C_V . However, the general behaviour near the critical point is clearly demonstrated, and at other temperatures the values should be reasonably accurate since the derivative $(\partial V/\partial p)_T$ does not change so rapidly with temperature. Despite the limited accuracy the results are of some interest since no other systematic data for C_p exist in this range.

6. CONCLUSIONS

Except for the measurements of C_p by Kramers *et al.* (1952) between 0.2 and 1.80 °K, and the density of liquid helium under its saturation vapour pressure, all the thermodynamic functions have been computed from measurements of the present authors only. No formulae other than general thermodynamic relations have been used for the calculations, and all important quantities entering them have been measured directly. Experiments have been performed throughout the stated range so that no reliance on interpolated or extrapolated values has been necessary. The experimental data especially at the higher densities are very considerable, so that the calculations have a much firmer basis than earlier ones.

If the present work is combined with that of Keesom, Bijl & Monté above 20 °K, the most important thermodynamic properties of helium are known to an accuracy of 1% in the whole temperature range from 3 to 500 °K and up to pressures of at least 100 atm. When new measurements by Lounasmaa & Kojo (1959) and Lounasmaa (1960) have been utilized, this range will be extended down to 1.5 °K.

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TABLE 1. EXPERIMENTAL POINTS OF C_V (J DEG⁻¹ MOLE⁻¹)

T (°K)	C_V	T (°K)	C_V	T (°K)	C_V	T (°K)	C_V
$\rho = 1.574$		$\rho = 2.256$ (cont.)		$\rho = 2.260$ (cont.)		$\rho = 2.136$ (cont.)	
4.12	16.61	4.59	8.96	4.98	9.25	6.56	10.34
4.29	18.20	5.14	9.35	5.33	9.50	7.05	10.59
4.45	19.08	5.40	9.58	5.73	9.77	7.60	10.82
4.61	19.92	5.70	9.73	6.19	10.04		
4.76	21.43	6.06	10.00	6.70	10.30	$\rho = 2.039$	
4.95	10.84	6.45	10.21	7.23	10.55	2.52	9.21
5.21	10.65	6.95	10.38	7.74	10.73	2.69	9.21
5.45	10.73	7.38	10.63	8.28	10.90	2.86	9.56
5.64	10.78	7.88	10.78	8.86	11.07	3.01	9.73
5.84	10.88	8.36	10.90	9.42	11.24	3.20	8.39
6.05	10.94	8.88	11.11	9.96	11.38	3.45	8.60
6.32	11.05			10.49	11.51	3.73	8.85
6.59	11.13	$\rho = 1.906$				4.00	9.10
6.86	11.22	2.955	10.00	$\rho = 1.427$		4.27	9.33
		3.165	10.65	4.72	22.93	4.60	9.58
$\rho = 1.263$		3.36	11.40	4.82	23.85	4.94	9.75
4.10	18.62	3.535	12.24	4.92	25.36	5.30	9.96
4.26	19.92	3.705	13.04	5.06	15.69	5.66	10.13
4.39	21.26	3.88	10.67	5.25	11.34	6.10	10.34
4.53	22.22	4.06	9.50	5.48	11.17	7.09	10.78
4.68	23.44	4.215	9.54	5.76	11.22	7.64	10.99
4.78	24.94	4.37	9.65	6.08	11.26	8.14	11.15
4.90	26.28	4.64	9.86	6.46	11.34		
5.01	28.62	4.95	9.98	6.91	11.43		
5.17	20.55	5.29	10.17	7.38	11.51	$\rho = 1.578$	
5.44	12.01	5.67	10.34	7.95	11.57	4.54	19.96
5.75	11.84	6.07	10.55	8.61	11.70	4.63	20.59
5.99	11.68	6.55	10.76	9.33	11.76	4.72	21.34
6.20	11.68	7.05	10.90			4.80	16.11
6.43	11.76			$\rho = 1.054$		4.93	10.55
		$\rho = 1.749$		4.82	29.34	5.08	10.59
$\rho = 1.292$		3.51	12.43	4.91	30.72	5.25	10.65
4.21	19.75	3.65	13.08	5.00	33.19	5.47	10.76
4.43	21.34	3.795	13.94	5.09	36.83	5.80	10.88
4.61	22.89	3.95	14.69	5.19	28.04	6.17	11.05
4.79	24.98	4.08	15.48	5.33	14.00	6.62	11.22
4.96	27.45	4.185	16.11	5.52	12.89	7.10	11.30
5.06	28.96	4.25	16.74	5.74	12.45	7.61	11.40
5.11	25.32	4.35	17.16	6.06	12.14		
5.19	12.33	4.49	10.30	6.49	12.09	$\rho = 2.564$	
5.43	11.80	4.87	10.17	6.96	12.01	2.10	4.52
5.56	11.68	5.05	10.25	7.45	12.01	2.37	4.83
5.73	11.53	5.56	10.46	8.02	12.01	2.64	5.17
5.97	11.51	5.86	10.59	8.73	12.01	2.89	5.55
6.32	11.55	6.17	10.73	9.48	12.03	3.13	5.90
6.79	11.59	6.54	10.88	10.31	12.03	3.345	6.21
		6.95	10.98			3.545	6.49
$\rho = 2.256$		7.44	11.15	$\rho = 2.136$		3.73	6.78
2.02	24.86	7.98	11.24	2.515	8.08	3.92	7.07
2.095	31.34	8.55	11.36	2.695	7.70	4.11	7.34
2.22	11.63			2.88	7.70	4.35	7.64
2.425	7.30	$\rho = 2.260$		3.06	7.83	4.63	7.91
2.62	6.91	2.47	6.99	3.25	7.97	4.92	8.24
2.80	6.95	2.84	6.95	3.50	8.20	5.16	8.41
2.97	7.05	3.03	7.07	3.77	8.52	5.41	8.64
3.15	7.26	3.21	7.26	4.04	8.79	5.66	8.91
3.33	7.43	3.38	7.45	4.31	9.08	6.01	9.21
3.51	7.68	3.58	7.78	4.62	9.33	6.41	9.48
3.71	7.97	3.80	8.06	4.96	9.50	6.75	9.69
3.99	8.35	4.04	8.33	5.32	9.73	7.15	9.94
4.28	8.70	4.31	8.64	5.66	9.94	7.59	10.21
		4.64	8.96	6.10	10.15	8.02	10.38

TABLE 1 (*cont.*)

T ($^{\circ}\text{K}$)	C_V	T ($^{\circ}\text{K}$)	C_V	T ($^{\circ}\text{K}$)	C_V	T ($^{\circ}\text{K}$)	C_V
$\rho = 2.564$ (<i>cont.</i>)		$\rho = 2.646$ (<i>cont.</i>)		$\rho = 1.138$		$\rho = 0.514$ (<i>cont.</i>)	
8.41	10.50	3.51	6.17	5.52	12.49	5.83	12.72
8.82	10.67	3.83	6.65	5.91	11.93	6.29	12.60
$\rho = 2.177$		4.16	7.01	6.33	11.84	6.90	12.43
14.15	12.12	4.52	7.39	6.91	11.82	7.64	12.26
16.00	12.24	4.97	7.91	7.74	11.82	8.48	12.30
17.74	12.53	5.47	8.39	8.80	11.84	9.45	12.28
$\rho = 1.587$		6.00	8.93	10.13	11.93	10.55	12.26
12.35	12.03	6.55	9.35	11.55	12.09	13.17	12.32
14.07	12.16	7.13	9.75	13.35	12.20	$\rho = 1.053$	
15.86	12.16	7.79	10.04	15.39	12.20	4.87	30.80
17.53	12.35	8.59	10.44	$\rho = 0.794$		4.96	32.68
19.28	12.68	9.64	10.86	5.45	13.45	5.04	34.94
$\rho = 1.082$		10.85	11.30	5.80	12.79	5.11	39.59
13.76	12.26	12.01	11.63	6.22	12.47	5.21	22.18
14.95	12.37	$\rho = 2.407$		6.78	12.22	5.37	13.60
16.46	12.43	2.43	5.90	7.56	12.14	5.60	12.76
18.16	12.51	2.61	5.94	8.65	12.09	5.84	12.51
19.85	12.62	2.83	6.11	10.09	12.16	6.05	12.30
$\rho = 1.870$		3.09	6.44	11.57	12.16	6.35	12.20
11.80	11.91	3.38	6.82	13.18	12.22	6.73	12.16
12.97	12.09	3.68	7.24	14.98	12.22	$\rho = 0.688$	
15.44	12.32	4.05	7.70	$\rho = 1.636$		5.00	48.38
16.78	12.43	4.44	8.14	5.56	10.63	5.05	51.06
19.26	12.56	4.93	8.58	5.94	10.80	5.16	16.82
$\rho = 2.896$		5.40	9.06	6.45	10.99	5.34	13.66
3.02	4.90	6.00	9.46	7.05	11.13	5.54	13.20
3.36	5.36	6.75	9.94	7.68	11.28	5.75	12.95
3.71	5.82	7.53	10.38	8.46	11.43	5.98	12.72
4.12	6.28	8.39	10.69	9.32	11.55	6.38	12.64
4.54	6.80	9.55	11.09	10.51	11.68	6.86	12.47
4.97	7.26	10.83	11.47	11.65	11.86	$\rho = 0.323$	
5.48	7.76	12.08	11.82	12.80	12.05	4.24	54.87
6.00	8.22	$\rho = 1.703$		13.96	12.12	4.29	56.29
6.55	8.64	4.63	13.62	$\rho = 1.113$		4.34	57.92
7.11	9.08	5.08	10.32	5.43	12.79	4.39	59.43
7.65	9.44	5.59	10.57	5.76	12.16	4.435	61.52
8.25	9.83	6.08	10.78	6.21	11.95	4.555	15.34
$\rho = 2.646$		6.59	10.92	6.78	11.91	4.745	12.79
2.22	4.44	7.15	11.07	7.49	11.86	4.94	12.72
2.46	4.62	7.88	11.24	8.38	11.84	5.16	12.56
2.69	5.00	8.74	11.38	9.43	11.97	5.44	12.47
2.93	5.34	9.80	11.59	$\rho = 0.514$		5.81	12.43
3.21	5.73	10.92	11.74	4.94	24.36	6.25	12.35
		12.09	11.93	5.15	13.29	6.65	12.35
		13.44	12.07	5.45	12.87		
		14.90	12.16				

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TABLE 2. THE SPECIFIC HEAT C_V (J DEG⁻¹ MOLE⁻¹)

T (°K)	reduced density															
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
2.50	12.47	—	—	—	—	—	—	—	—	—	—	7.47	5.92	4.88	4.40	—
2.75	12.47	—	—	—	—	—	—	—	—	—	—	7.24	6.07	5.22	4.72	—
3.00	12.47	—	—	—	—	—	—	—	—	—	—	7.40	6.36	5.58	5.06	—
3.25	12.47	—	—	—	—	—	—	—	—	—	—	7.65	6.71	5.96	5.40	—
3.50	12.47	—	—	—	—	—	—	—	—	—	8.81	7.93	7.09	6.34	5.74	4.98
3.75	12.47	—	—	—	—	—	—	—	—	—	9.01	8.22	7.44	6.70	6.07	5.54
4.00	12.47	—	—	—	—	—	—	—	—	—	9.20	8.49	7.75	7.04	6.39	5.82
4.25	12.47	12.73	—	—	—	—	—	—	—	—	9.38	8.73	8.04	7.35	6.70	6.10
4.50	12.47	12.67	—	—	—	—	—	—	—	10.01	9.54	8.95	8.30	7.64	6.99	6.37
4.75	12.47	12.61	—	—	—	—	—	—	—	10.12	9.69	9.15	8.54	7.91	7.27	6.64
5.00	12.47	12.56	13.08	—	—	—	—	—	10.66	10.23	9.83	9.33	8.76	8.16	7.54	6.90
5.25	12.47	12.52	12.72	—	—	—	—	11.39	10.68	10.34	9.96	9.50	8.97	8.40	7.79	(7.15)
5.50	12.47	12.48	12.59	12.87	13.01	12.74	12.00	11.22	10.75	10.45	10.09	9.66	9.16	8.62	8.03	(7.40)
6.00	12.47	12.43	12.44	12.53	12.46	12.12	11.71	11.28	10.95	10.65	10.33	9.95	9.51	9.03	8.48	(7.87)
6.50	12.47	12.40	12.37	12.33	12.19	11.96	11.70	11.42	11.13	10.84	10.54	10.20	9.82	9.38	8.89	(8.32)
7.00	12.47	12.40	12.34	12.25	12.12	11.95	11.74	11.52	11.27	11.01	10.73	10.42	10.08	9.70	9.26	(8.74)
7.50	12.47	12.40	12.32	12.22	12.10	11.95	11.78	11.59	11.37	11.14	10.89	10.62	10.32	9.99	9.60	(9.13)
8.00	12.47	12.40	12.32	12.22	12.10	11.97	11.82	11.65	11.46	11.26	11.03	10.79	10.53	10.24	9.90	(9.49)
9.00	12.47	12.39	12.31	12.22	12.12	12.01	11.89	11.76	11.61	11.45	11.28	11.10	10.90	10.69	10.44	(10.14)
10.00	12.47	12.39	12.31	12.23	12.14	12.05	11.95	11.85	11.73	11.61	11.49	11.35	11.21	11.06	(10.89)	(10.69)
11.00	12.47	12.40	12.33	12.25	12.18	12.10	12.02	11.94	11.85	11.76	11.67	11.58	11.48	11.37	(11.26)	(11.09)
12.00	12.47	12.40	12.34	12.28	12.22	12.16	12.10	12.04	11.97	11.90	11.83	11.77	11.70	11.63	(11.56)	(11.49)
13.00	12.47	12.41	12.36	12.31	12.27	12.22	12.18	12.13	12.08	12.03	11.98	11.94	11.89	11.84	(11.80)	(11.76)
14.00	12.47	12.43	12.39	12.35	12.31	12.28	12.24	12.21	12.18	12.15	12.12	12.09	12.06	(12.03)	(12.01)	(11.99)
15.00	12.47	12.44	12.41	12.38	12.36	12.34	12.32	12.30	12.28	12.26	12.24	12.22	12.20	(12.18)	(12.17)	(12.16)
16.00	12.47	12.45	12.43	12.41	12.40	12.39	12.38	12.37	12.36	12.35	12.34	12.33	12.33	(12.32)	(12.31)	(12.31)
17.00	12.47	12.46	12.45	12.44	12.44	12.44	12.44	12.44	12.44	12.44	12.44	12.44	(12.44)	(12.44)	(12.44)	(12.45)
18.00	12.47	12.47	12.47	12.47	12.48	12.49	12.50	12.51	12.51	12.52	12.53	12.54	(12.55)	(12.55)	(12.56)	(12.57)
19.00	12.47	12.48	12.49	12.50	12.52	12.53	12.55	12.57	12.58	12.59	12.61	12.62	(12.64)	(12.65)	(12.67)	(12.68)
20.00	12.47	12.49	12.51	12.53	12.55	12.57	12.59	12.62	12.64	12.66	12.68	(12.70)	(12.72)	(12.74)	(12.76)	(12.78)

TABLE 3. EXPERIMENTAL POINTS OF p (ATM)

T ($^{\circ}\text{K}$)	p	T ($^{\circ}\text{K}$)	p	T ($^{\circ}\text{K}$)	p	T ($^{\circ}\text{K}$)	p
$\rho = 1.468$		$\rho = 0.4877$ (cont.)		$\rho = 1.120$ (cont.)		$\rho = 2.832$ (cont.)	
11.19	21.04	9.79	5.68	10.31	13.07	4.420	67.6
12.05	23.82	10.73	6.43	11.50	15.70	4.75	69.8
12.91	26.28	12.11	7.53	$\rho = 1.602$			
13.79	29.28	13.67	8.75	10.59	22.42	5.51	75.2
14.69	32.27	$\rho = 2.310$		12.25	28.46	5.85	77.6
15.62	35.16	3.610	13.02	13.98	34.87	6.27	80.4
16.55	38.11	3.859	14.15	15.81	41.60	6.95	85.8
17.51	41.24	4.168	15.62	17.78	48.9	7.67	91.3
18.52	44.42	4.398	16.78	19.69	56.0	8.36	96.9
19.34	47.10	4.70	18.34	$\rho = 1.077$			
$\rho = 0.873$		4.92	19.51	10.27	12.61	$\rho = 2.749$	
10.50	10.46	5.20	21.11	11.64	15.41	3.051	49.3
12.03	12.83	5.64	23.47	13.03	18.30	3.256	50.3
13.69	15.46	6.12	26.39	14.43	21.28	3.500	51.5
15.41	18.17	6.64	29.38	15.89	24.34	3.692	52.5
17.13	20.87	7.15	32.49	17.38	27.43	4.000	54.2
18.86	23.59	7.63	35.56	18.84	30.53	4.284	55.8
20.14	25.64	8.15	38.83	20.23	33.47	4.517	57.2
$\rho = 0.5173$		9.19	45.52	$\rho = 2.923$			
10.60	6.65	10.24	52.6	3.262	75.0	4.83	59.5
11.98	7.83	11.32	59.7	3.428	76.0	5.18	61.8
13.56	9.14	$\rho = 1.952$		3.638	77.2	5.61	64.2
14.92	10.31	3.822	1.35	3.837	78.5	5.93	66.5
16.25	11.43	4.116	2.57	4.150	80.2	6.35	69.6
17.94	12.83	4.400	3.80	4.370	81.7	7.05	75.0
20.07	14.67	4.67	5.01	4.62	83.4	7.76	80.2
$\rho = 1.435$		4.96	6.31	4.95	85.6	8.45	85.6
5.17	2.33	5.47	8.73	5.29	88.0	9.12	91.1
5.56	3.44	6.08	11.50	5.72	91.2	10.42	101.8
6.01	4.73	6.63	14.11	6.04	93.6	$\rho = 2.085$	
6.41	5.88	7.16	16.67	6.46	96.7	10.52	39.8
6.86	7.20	7.76	19.46	7.07	101.5	12.29	49.4
7.67	9.62	8.32	22.30	$\rho = 2.748$			
8.48	12.13	8.93	25.28	3.083	49.2	14.07	59.5
9.32	14.68	10.12	31.19	3.290	50.2	15.82	69.4
10.14	17.26	11.31	37.15	3.533	51.5	17.79	80.3
11.02	20.00	$\rho = 1.683$		3.772	52.8	19.50	89.9
11.91	22.74	4.306	1.10	4.023	54.2	$\rho = 1.869$	
12.70	25.21	4.86	2.39	4.300	55.8	13.48	44.0
$\rho = 0.811$		5.19	3.57	4.532	57.2	15.62	54.0
5.08	2.09	5.50	4.70	4.83	59.3	17.78	64.1
5.63	2.85	5.86	6.03	5.19	61.7	19.89	74.1
6.25	3.72	6.55	8.61	5.62	64.2	$\rho = 0.2882$	
6.82	4.52	7.26	11.30	5.94	66.5	9.45	3.43
7.65	5.69	8.00	14.11	6.36	69.5	11.73	4.43
8.54	6.97	8.70	16.84	7.07	74.9	14.26	5.55
9.34	8.11	9.42	19.64	7.78	80.1	16.59	6.57
10.26	9.47	10.14	22.54	8.45	85.5	18.97	7.64
11.06	10.64	11.61	28.36	9.13	90.9	21.38	8.70
12.10	12.12	$\rho = 1.120$		10.42	101.6	$\rho = 2.521$	
13.77	14.52	4.424	1.21	$\rho = 2.832$			
$\rho = 0.4877$		5.03	2.02	2.976	59.6	10.75	76.1
4.63	1.46	5.59	3.06	3.192	60.7	12.22	86.7
5.15	2.00	5.93	3.74	3.427	61.9	13.60	97.1
6.04	2.72	6.51	4.91	3.630	63.0	$\rho = 2.385$	
6.78	3.29	7.13	6.22	3.928	64.5	10.34	59.4
7.62	3.95	7.94	7.93	4.191	66.1	11.85	69.6
8.47	4.62	9.13	10.48	$\rho = 2.835$			
						13.40	80.3
						14.79	90.0
						16.30	100.6

TABLE 3 (cont.)

T (°K)	p	T (°K)	p	T (°K)	p	T (°K)	p
$\rho = 2.216$		$\rho = 2.573$ (cont.)		$\rho = 2.188$ (cont.)		$\rho = 1.189$	
12.17	58.2	5.05	40.06	3.747	7.84	4.430	1.21
13.87	68.3	5.45	42.56	3.989	8.93	5.12	2.15
15.54	79.1	5.79	44.67	4.304	10.43	5.70	3.35
17.34	90.0	6.18	47.5	4.595	11.86	6.26	4.59
19.08	100.6	6.91	52.2	4.88	13.35	6.66	5.52
$\rho = 2.573$		7.62	57.6	5.29	15.58	7.32	7.01
2.857	28.71	8.32	62.4	5.73	17.77	8.01	8.60
3.010	29.34	9.01	67.7	6.10	19.93	8.56	9.91
3.354	30.84	10.41	78.2	6.53	22.25	9.47	12.00
3.611	32.05	11.75	88.5	7.34	26.81	10.40	14.22
3.865	33.33	$\rho = 2.188$		8.12	31.36	12.22	18.58
4.130	34.72	2.948	4.63	8.91	36.05		
4.392	36.17	3.233	5.69	10.55	45.64		
4.69	37.92	3.494	6.75	12.15	55.8		
				13.79	65.6		

TABLE 4. THE PRESSURE p (ATM)

$T(^{\circ}\text{K})$	reduced density																			
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3.00	0.00	—	—	—	—	—	—	—	—	—	—	5.23	9.85	15.78	23.00	31.70	42.05	54.55	69.50	—
3.25	0.00	—	—	—	—	—	—	—	—	—	—	6.24	10.90	16.83	24.10	32.85	43.25	55.85	70.85	89.25
3.50	0.00	—	—	—	—	—	—	—	—	—	1.12	7.29	11.98	17.94	25.25	34.05	44.55	57.20	72.30	90.75
3.75	0.00	—	—	—	—	—	—	—	—	—	2.12	8.38	13.11	19.11	26.45	35.30	45.90	58.60	73.80	92.30
4.00	0.00	—	—	—	—	—	—	—	—	—	3.16	9.51	14.28	20.33	27.70	36.60	47.30	60.05	75.35	93.95
4.25	0.00	0.92	—	—	—	—	—	—	—	—	4.22	10.67	15.49	21.59	29.00	37.95	48.75	61.60	76.95	95.65
4.50	0.00	0.99	—	—	—	—	—	—	—	1.99	5.31	11.87	16.74	22.90	30.35	39.40	50.25	63.20	78.60	97.40
4.75	0.00	1.58	—	—	—	—	—	—	—	2.99	6.43	13.10	18.03	24.25	31.75	40.85	51.80	64.85	80.30	99.20
5.00	0.00	1.14	1.74	—	—	—	—	—	2.36	4.01	7.58	14.37	19.35	25.60	33.20	42.35	53.40	66.55	82.05	101.05
5.25	0.00	1.21	1.90	—	—	—	—	—	3.19	5.04	8.76	15.67	20.71	27.05	34.70	43.90	55.05	68.25	83.85	—
5.50	0.00	1.28	2.05	—	—	2.40	2.93	—	4.03	6.08	9.96	16.99	22.10	28.50	36.25	45.50	56.75	70.00	85.70	—
6.00	0.00	1.43	2.37	—	—	2.80	3.68	—	5.75	8.16	12.41	19.72	24.97	31.50	39.40	48.85	60.20	73.65	89.50	—
6.50	0.00	1.58	2.69	—	—	3.68	4.58	—	7.50	10.26	14.90	22.54	27.95	34.60	42.70	52.35	63.85	77.45	93.45	—
7.00	0.00	1.73	3.00	—	—	4.58	5.15	—	9.27	12.39	17.43	25.45	31.00	37.80	46.10	55.95	67.60	81.35	97.50	—
7.50	0.00	1.87	3.32	—	—	5.48	6.39	—	11.04	14.53	19.97	28.40	34.10	41.10	49.55	59.60	71.45	85.35	101.65	—
8.00	0.00	2.03	3.64	—	—	6.12	7.30	—	12.82	16.69	22.55	31.35	37.25	44.45	53.10	63.30	75.35	89.40	—	—
9.00	0.00	2.32	4.27	—	—	7.52	9.15	—	16.44	21.03	27.75	37.40	43.70	51.30	60.30	70.85	83.35	97.75	—	—
10.00	0.00	2.62	4.90	—	—	8.93	10.98	—	20.10	25.40	32.95	43.50	50.25	58.25	67.60	78.55	91.55	—	—	—
11.00	0.00	2.92	5.53	—	—	10.35	12.89	—	23.78	29.80	38.20	49.65	56.85	65.25	75.00	86.45	99.85	—	—	—
12.00	0.00	3.22	6.15	—	—	11.77	14.78	—	27.45	34.25	43.40	55.75	63.40	72.25	82.55	94.50	—	—	—	—
13.00	0.00	3.51	6.78	—	—	13.19	16.68	—	31.20	38.65	48.65	61.85	70.00	79.35	90.20	102.65	—	—	—	—
14.00	0.00	3.80	7.42	—	—	14.61	18.58	—	34.90	43.10	53.90	68.00	76.60	86.45	—	—	—	—	—	—
15.00	0.00	4.10	8.04	—	—	16.03	20.50	—	38.60	47.55	59.10	74.15	83.20	93.55	—	—	—	—	—	—
16.00	0.00	4.40	8.68	—	—	17.44	22.41	—	42.30	51.95	64.35	80.30	89.80	100.65	—	—	—	—	—	—
17.00	0.00	4.70	9.30	—	—	18.85	24.31	—	46.00	56.40	69.60	86.50	96.45	—	—	—	—	—	—	—
18.00	0.00	5.00	9.93	—	—	20.26	26.20	—	49.70	60.85	74.80	92.65	103.05	—	—	—	—	—	—	—
19.00	0.00	5.30	10.57	—	—	21.68	28.05	—	53.45	65.30	80.05	98.80	—	—	—	—	—	—	—	—
20.00	0.00	5.60	11.21	—	—	23.10	29.95	—	57.20	69.75	85.25	—	—	—	—	—	—	—	—	—

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TABLE 5. EXPERIMENTAL POINTS OF $(\partial p/\partial T)_V$ (ATM DEG⁻¹)

T (°K)	$(\partial p/\partial T)_V$	T (°K)	$(\partial p/\partial T)_V$	T (°K)	$(\partial p/\partial T)_V$	T (°K)	$(\partial p/\partial T)_V$
$\rho = 1.468$		$\rho = 2.310$ (cont.)		$\rho = 2.923$ (cont.)		$\rho = 0.2882$	
11.59	3.16	6.86	6.16	5.88	7.82	10.59	0.442
12.42	3.23	7.37	6.29	6.20	7.90	13.00	0.440
13.29	3.25	7.84	6.36	6.77	8.21	15.43	0.440
14.18	3.24	8.63	6.44			17.77	0.444
15.09	3.26	9.66	6.53	$\rho = 2.748$		20.17	0.444
16.01	3.25	10.78	6.59	3.186	5.04		
16.96	3.23			3.411	5.21	$\rho = 2.521$	
17.93	3.23	$\rho = 1.952$		3.652	5.48	11.44	7.58
18.93	3.24	3.969	4.18	3.863	5.60	12.91	7.66
		4.253	4.32	4.161	5.94	$\rho = 2.385$	
$\rho = 0.873$		4.535	4.43	4.416	6.19	11.05	6.92
11.21	1.57	4.82	4.50	4.654	6.39	12.58	6.96
12.79	1.575	5.21	4.62	5.01	6.66	14.04	7.06
14.48	1.57	5.76	4.71	5.37	6.87	15.55	7.02
16.18	1.57	6.34	4.80	5.78	7.14	$\rho = 2.216$	
17.90	1.59	6.88	4.87	6.11	7.33	12.97	6.21
19.50	1.60	7.43	4.90	6.67	7.61	14.66	6.20
		8.01	4.94	7.38	7.87	16.38	6.25
$\rho = 0.5173$		8.59	4.97	8.08	8.09	18.21	6.26
11.29	0.855	9.49	5.00	8.74	8.26		
12.71	0.839	10.71	5.02	9.78	8.48	$\rho = 2.573$	
14.21	0.837					2.934	4.12
15.58	0.838	$\rho = 1.683$		$\rho = 2.832$		3.182	4.37
17.00	0.843	5.01	3.61	3.084	5.23	3.482	4.79
19.00	0.855	5.34	3.69	3.309	5.41	3.714	5.03
		5.67	3.71	3.528	5.60	3.998	5.30
$\rho = 1.435$		6.20	3.78	3.734	5.87	4.261	5.57
5.77	2.87	6.88	3.85	4.059	6.13	4.519	5.80
6.21	2.96	7.61	3.88	4.305	6.40	4.87	6.04
6.62	2.97	8.32	3.91	4.547	6.54	5.23	6.28
7.24	3.01	9.02	3.94	4.91	6.85	5.62	6.50
8.06	3.04	9.75	3.95	5.25	7.08	5.95	6.69
8.87	3.07	10.87	3.97	5.68	7.39	6.51	6.96
9.70	3.08			6.00	7.53	7.24	7.17
10.54	3.09	$\rho = 1.120$		6.57	7.84	7.94	7.39
11.41	3.08	5.75	2.02	7.27	8.13	8.63	7.52
12.30	3.11	6.22	2.08	7.97	8.39	9.67	7.70
		6.80	2.11	8.65	8.59	11.08	7.79
		7.51	2.14	$\rho = 2.749$		$\rho = 2.188$	
$\rho = 0.811$		8.50	2.16	3.154	5.05	3.091	3.73
5.94	1.41	9.68	2.18	3.378	5.21	3.363	4.08
6.51	1.42	10.90	2.18	3.596	5.36	3.620	4.32
7.23	1.425			3.807	5.62	3.859	4.56
8.06	1.43	$\rho = 1.602$		4.142	5.93	4.147	4.74
8.94	1.435	11.36	3.70	4.400	6.19	4.449	4.97
9.77	1.44	13.05	3.69	4.646	6.43	4.732	5.12
10.66	1.44	14.81	3.70	5.01	6.66	5.08	5.29
11.53	1.44	16.71	3.72	5.36	6.98	5.49	5.40
12.93	1.435	18.73	3.73	5.76	7.18	5.91	5.53
				6.09	7.28	6.30	5.66
$\rho = 0.4877$		$\rho = 1.077$		6.66	7.63	6.91	5.74
6.41	0.802	10.90	2.10	7.37	7.87	7.71	5.85
7.17	0.792	12.27	2.10	8.06	8.12	8.49	5.94
8.04	0.790	13.67	2.11	8.75	8.28	9.71	6.01
9.13	0.794	15.09	2.09	9.77	8.43	11.33	6.13
10.21	0.785	16.59	2.12	$\rho = 2.085$		12.97	6.07
11.42	0.788	18.02	2.12	11.37	5.55	$\rho = 1.189$	
12.89	0.787	19.53	2.13	13.13	5.61	5.98	2.23
				14.95	5.59	6.45	2.26
$\rho = 2.310$		$\rho = 2.923$		16.74	5.64	6.99	2.29
3.734	4.57	3.533	6.00	18.65	5.68	7.66	2.32
3.993	4.87	3.737	6.19	$\rho = 1.869$		8.27	2.36
4.283	5.08	3.942	6.36	14.50	4.70	9.02	2.36
4.53	5.28	4.260	6.61	16.64	4.74	9.91	2.36
4.81	5.45	4.495	6.81	18.84	4.77	11.34	2.37
5.05	5.59	4.73	7.03				
5.40	5.74	5.12	7.34				
5.85	5.90	5.46	7.47				
6.34	6.05						

TABLE 6. $(\partial p / \partial T)_V$ (ATM DEG⁻¹)
reduced density

T (°K)	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3.00	0.00	—	—	—	—	—	—	—	—	—	—	3.68	3.79	3.94	4.13	4.36	4.64	4.95	5.29	—
3.25	0.00	—	—	—	—	—	—	—	—	—	—	3.96	4.08	4.23	4.42	4.65	4.93	5.24	5.58	5.95
3.50	0.00	—	—	—	—	—	—	—	—	—	3.96	4.21	4.34	4.50	4.69	4.92	5.20	5.51	5.85	6.23
3.75	0.00	—	—	—	—	—	—	—	—	—	4.14	4.43	4.58	4.74	4.94	5.17	5.45	5.77	6.11	6.49
4.00	0.00	—	—	—	—	—	—	—	—	—	4.29	4.63	4.79	4.97	5.17	5.41	5.69	6.01	6.35	6.73
4.25	0.00	0.30	—	—	—	—	—	—	—	—	4.42	4.80	4.98	5.18	5.39	5.63	5.92	6.23	6.57	6.96
4.50	0.00	0.30	—	—	—	—	—	—	—	—	4.53	4.96	5.16	5.37	5.59	5.84	6.13	6.44	6.78	7.17
4.75	0.00	0.30	0.62	—	—	—	—	—	—	4.01	4.62	5.10	5.32	5.54	5.77	6.03	6.33	6.64	6.98	7.36
5.00	0.00	0.30	0.62	—	—	—	—	—	3.36	4.08	4.70	5.23	5.46	5.70	5.94	6.21	6.51	6.82	7.16	7.54
5.25	0.00	0.30	0.62	0.98	—	—	—	—	3.39	4.11	4.77	5.34	5.59	5.84	6.09	6.37	6.68	6.99	7.33	—
5.50	0.00	0.30	0.62	0.98	1.34	1.76	2.21	2.74	3.42	4.14	4.83	5.44	5.71	5.97	6.23	6.52	6.83	7.15	7.49	—
6.00	0.00	0.30	0.63	0.99	1.38	1.80	2.24	2.77	3.47	4.19	4.93	5.60	5.90	6.19	6.48	6.78	7.10	7.43	7.78	—
6.50	0.00	0.30	0.63	0.99	1.39	1.82	2.31	2.85	3.51	4.23	5.01	5.72	6.05	6.37	6.68	7.00	7.34	7.68	8.04	—
7.00	0.00	0.30	0.63	0.99	1.40	1.83	2.33	2.88	3.54	4.27	5.06	5.81	6.17	6.51	6.85	7.19	7.54	7.90	8.27	—
8.00	0.00	0.30	0.63	0.99	1.40	1.85	2.36	2.93	3.59	4.32	5.13	5.93	6.33	6.72	7.10	7.48	7.86	8.25	—	—
9.00	0.00	0.30	0.63	0.99	1.40	1.86	2.38	2.96	3.62	4.35	5.17	6.00	6.43	6.86	7.28	7.70	8.12	8.54	—	—
10.00	0.00	0.30	0.63	0.99	1.40	1.87	2.39	2.98	3.64	4.37	5.20	6.05	6.50	6.95	7.40	7.86	8.32	—	—	—
11.00	0.00	0.30	0.63	0.99	1.40	1.87	2.41	3.00	3.66	4.40	5.22	6.09	6.55	7.01	7.48	7.97	8.47	—	—	—
12.00	0.00	0.30	0.63	0.99	1.40	1.88	2.41	3.01	3.66	4.40	5.23	6.12	6.58	7.05	7.54	8.05	—	—	—	—
14.00	0.00	0.30	0.63	0.99	1.40	1.88	2.41	3.01	3.67	4.42	5.25	6.16	6.62	7.11	7.61	—	—	—	—	—
16.00	0.00	0.30	0.63	0.99	1.41	1.89	2.42	3.02	3.69	4.44	5.27	6.18	6.65	7.16	—	—	—	—	—	—
18.00	0.00	0.30	0.63	0.99	1.41	1.89	2.42	3.03	3.71	4.46	5.29	6.20	6.67	—	—	—	—	—	—	—
20.00	0.00	0.30	0.63	0.99	1.41	1.89	2.43	3.04	3.73	4.48	5.31	—	—	—	—	—	—	—	—	—

THERMODYNAMIC PROPERTIES OF FLUID HELIUM

TABLE 7. $-(\partial V/\partial p)_T$ (CM³ ATM⁻¹ MOLE⁻¹)

T (°K)	reduced density																			
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3.00	∞	—	—	—	—	—	—	—	—	—	—	0.303	0.209	0.153	0.117	0.0903	0.0698	0.0540	0.0416	—
3.25	∞	—	—	—	—	—	—	—	—	—	—	0.298	0.208	0.153	0.116	0.0900	0.0694	0.0538	0.0414	0.0323
3.50	∞	—	—	—	—	—	—	—	—	—	—	0.286	0.207	0.152	0.116	0.0891	0.0689	0.0535	0.0413	0.0321
3.75	∞	—	—	—	—	—	—	—	—	—	—	0.283	0.206	0.151	0.115	0.0885	0.0685	0.0532	0.0411	0.0320
4.00	∞	—	—	—	—	—	—	—	—	—	—	0.280	0.204	0.151	0.115	0.0878	0.0680	0.0529	0.0408	0.0318
4.25	∞	—	—	—	—	—	—	—	—	—	—	0.277	0.202	0.149	0.114	0.0871	0.0675	0.0526	0.0407	0.0317
4.50	∞	—	—	—	—	—	—	—	—	—	0.589	0.273	0.199	0.148	0.113	0.0865	0.0671	0.0524	0.0405	0.0315
4.75	∞	—	—	—	—	—	—	—	—	—	0.575	0.269	0.197	0.147	0.112	0.0858	0.0665	0.0521	0.0403	0.0314
5.00	∞	—	—	—	—	—	—	—	—	1.38	0.562	0.267	0.195	0.146	0.111	0.0852	0.0660	0.0518	0.0401	0.0312
5.25	∞	334	—	—	—	—	—	—	—	1.29	0.547	0.263	0.193	0.144	0.111	0.0845	0.0656	0.0516	0.0398	—
5.50	∞	306	144	151	246	342	101	15.0	3.62	1.29	0.534	0.259	0.190	0.143	0.110	0.0839	0.0652	0.0513	0.0396	—
5.75	∞	284	123	105	107	89.4	40.4	10.8	3.15	1.21	0.504	0.252	0.186	0.140	0.108	0.0827	0.0644	0.0507	0.0393	—
6.00	∞	246	95.2	65.4	50.5	34.7	17.9	6.90	2.54	1.08	0.473	0.244	0.182	0.137	0.105	0.0813	0.0636	0.0502	0.0389	—
6.50	∞	217	77.4	47.5	32.4	21.2	11.5	5.06	2.13	0.970	0.446	0.236	0.178	0.134	0.103	0.0801	0.0629	0.0497	0.0387	—
7.00	∞	194	65.3	37.2	23.9	15.0	8.38	3.98	1.82	0.880	0.446	0.236	0.178	0.134	0.103	0.0801	0.0629	0.0497	0.0387	—
8.00	∞	160	49.6	26.1	15.5	9.31	5.39	2.88	1.42	0.738	0.398	0.220	0.168	0.128	0.0988	0.0774	0.0611	0.0489	—	—
9.00	∞	136	40.0	19.9	11.4	6.73	3.94	2.18	1.17	0.635	0.356	0.205	0.158	0.122	0.0953	0.0747	0.0593	0.0481	—	—
10.00	∞	119	33.6	16.0	8.95	5.24	3.08	1.76	0.992	0.559	0.321	0.190	0.149	0.117	0.0917	0.0718	0.0570	—	—	—
11.00	∞	105	28.9	13.4	7.36	4.28	2.54	1.48	0.861	0.499	0.293	0.178	0.141	0.112	0.0878	0.0692	0.0554	—	—	—
12.00	∞	94.6	25.4	11.5	6.24	3.62	2.15	1.29	0.761	0.450	0.271	0.167	0.133	0.106	0.0836	0.0672	—	—	—	—
14.00	∞	78.4	20.3	8.99	4.76	2.76	1.65	1.00	0.615	0.378	0.234	0.148	0.119	0.0946	0.0756	—	—	—	—	—
16.00	∞	67.0	17.0	7.37	3.85	2.21	1.34	0.831	0.520	0.325	0.206	0.132	0.108	0.0878	—	—	—	—	—	—
18.00	∞	58.6	14.6	6.27	3.23	1.84	1.13	0.705	0.446	0.286	0.183	0.119	0.100	—	—	—	—	—	—	—
20.00	∞	52.0	12.8	5.44	2.80	1.60	0.971	0.606	0.393	0.256	0.163	—	—	—	—	—	—	—	—	—

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

TABLE 8. THE COMPRESSIBILITY FACTOR pV/RT

T (°K)	reduced density																			
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3.00	1.000	—	—	—	—	—	—	—	—	—	—	0.562	1.012	1.553	2.175	2.881	3.681	4.603	5.663	—
3.25	1.000	—	—	—	—	—	—	—	—	—	—	0.619	1.034	1.529	2.103	2.756	3.495	4.351	5.329	6.490
3.50	1.000	—	—	—	—	—	—	—	—	—	0.113	0.671	1.055	1.514	2.046	2.653	3.343	4.139	5.049	6.127
3.75	1.000	—	—	—	—	—	—	—	—	—	0.200	0.720	1.078	1.505	2.000	2.567	3.215	3.958	4.813	5.817
4.00	1.000	—	—	—	—	—	—	—	—	—	0.280	0.766	1.101	1.501	2.004	2.495	3.105	3.803	4.607	5.552
4.25	1.000	0.767	—	—	—	—	—	—	—	—	0.352	0.809	1.124	1.500	1.935	2.435	3.011	3.672	4.428	5.317
4.50	1.000	0.780	—	—	—	—	—	—	—	0.174	0.418	0.850	1.147	1.502	1.913	2.387	2.933	3.558	4.271	5.115
4.75	1.000	0.798	—	—	—	—	—	—	—	0.248	0.480	0.889	1.170	1.507	1.896	2.345	2.863	3.458	4.134	4.937
5.00	1.000	0.808	0.617	—	—	—	—	—	0.209	0.316	0.537	0.926	1.193	1.512	1.883	2.310	2.806	3.371	4.012	4.776
5.25	1.000	0.817	0.641	0.500	—	—	—	—	0.269	0.378	0.591	0.962	1.216	1.521	1.875	2.279	2.710	3.223	3.809	—
5.50	1.000	0.825	0.661	0.529	0.430	0.361	0.270	0.244	0.325	0.435	0.642	0.996	1.239	1.530	1.869	2.256	2.635	3.108	3.648	—
6.00	1.000	0.845	0.700	0.583	0.496	0.435	0.397	0.387	0.425	0.536	0.733	1.060	1.283	1.550	1.862	2.220	2.635	3.108	3.648	—
6.50	1.000	0.861	0.734	0.629	0.552	0.500	0.468	0.467	0.511	0.622	0.812	1.117	1.326	1.571	1.863	2.197	2.580	3.016	3.516	—
7.00	1.000	0.876	0.760	0.669	0.600	0.555	0.531	0.536	0.587	0.697	0.883	1.171	1.365	1.595	1.868	2.180	2.537	2.943	3.406	—
8.00	1.000	0.899	0.806	0.733	0.678	0.647	0.637	0.652	0.710	0.822	0.999	1.263	1.436	1.641	1.883	2.158	2.474	2.829	—	—
9.00	1.000	0.914	0.841	0.781	0.741	0.721	0.721	0.746	0.809	0.920	1.093	1.340	1.496	1.683	1.900	2.148	2.432	2.750	—	—
10.00	1.000	0.929	0.868	0.821	0.792	0.781	0.790	0.823	0.891	1.000	1.168	1.402	1.549	1.720	1.917	2.143	2.405	—	—	—
11.00	1.000	0.941	0.891	0.854	0.834	0.831	0.848	0.887	0.958	1.067	1.231	1.455	1.593	1.751	1.933	2.143	2.384	—	—	—
12.00	1.000	0.951	0.908	0.881	0.869	0.873	0.897	0.941	1.013	1.124	1.282	1.497	1.629	1.778	1.951	2.148	—	—	—	—
14.00	1.000	0.962	0.939	0.924	0.925	0.941	0.973	1.025	1.105	1.213	1.365	1.565	1.687	1.823	1.984	—	—	—	—	—
16.00	1.000	0.974	0.961	0.957	0.966	0.993	1.034	1.090	1.171	1.279	1.425	1.618	1.731	1.857	—	—	—	—	—	—
18.00	1.000	0.985	0.978	0.981	0.998	1.031	1.078	1.139	1.224	1.332	1.474	1.659	1.765	—	—	—	—	—	—	—
20.00	1.000	0.993	0.993	1.001	1.024	1.062	1.112	1.118	1.267	1.374	1.511	—	—	—	—	—	—	—	—	—

THERMODYNAMIC PROPERTIES OF FLUID HELIUM

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TABLE 9. THE ENTROPY S ($J \text{ DEG}^{-1} \text{ MOLE}^{-1}$)

T (°K)	reduced density															
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
2.50	∞	—	—	—	—	—	—	—	—	—	—	7.408	6.803	6.203	5.569	—
2.75	∞	—	—	—	—	—	—	—	—	—	—	8.110	7.372	6.683	6.005	—
3.00	∞	—	—	—	—	—	—	—	—	—	—	8.740	7.913	7.153	6.430	—
3.25	∞	—	—	—	—	—	—	—	—	—	—	9.342	8.436	7.614	6.848	6.111
3.50	∞	—	—	—	—	—	—	—	—	—	11.01	9.919	8.947	8.070	7.261	6.494
3.75	∞	—	—	—	—	—	—	—	—	—	11.62	10.48	9.449	8.520	7.668	6.867
4.00	∞	35.54	—	—	—	—	—	—	—	—	12.21	11.02	9.939	8.963	8.070	7.233
4.25	∞	36.27	—	—	—	—	—	—	—	—	12.77	11.54	10.42	9.400	8.467	7.595
4.50	∞	36.95	—	—	—	—	—	—	—	14.69	13.31	12.04	10.89	9.828	8.858	7.951
4.75	∞	37.60	30.66	—	—	—	—	—	—	15.23	13.83	12.53	11.34	10.25	9.243	8.303
5.00	∞	37.60	31.33	—	—	—	—	—	17.25	15.75	14.33	13.01	11.78	10.66	9.623	8.650
5.25	∞	38.21	31.95	28.06	—	22.92	21.03	19.35	17.77	16.26	14.82	13.46	12.22	11.06	9.997	(8.993)
5.50	∞	38.79	32.54	28.67	25.82	23.53	21.60	19.87	18.27	16.74	15.28	13.91	12.64	11.46	10.36	(9.331)
6.00	∞	39.87	33.63	29.77	26.92	24.60	22.63	20.85	19.21	17.66	16.17	14.76	13.45	12.23	11.08	(9.995)
6.50	∞	40.87	34.62	30.77	27.91	25.57	23.56	21.76	20.10	18.52	17.01	15.57	14.22	12.97	11.78	(10.65)
7.00	∞	41.79	35.54	31.68	28.81	26.45	24.43	22.61	20.93	19.33	17.80	16.33	14.96	13.67	12.45	(11.28)
7.50	∞	42.64	36.39	32.52	29.64	27.28	25.25	23.41	21.71	20.09	18.54	17.06	15.67	14.35	13.10	(11.90)
8.00	∞	43.44	37.18	33.31	30.42	28.05	26.01	24.16	22.45	20.81	19.25	17.75	16.34	15.00	13.73	(12.50)
9.00	∞	44.90	38.64	34.75	31.85	29.46	27.40	25.54	23.81	22.15	20.56	19.04	17.60	16.24	14.93	(13.65)
10.00	∞	46.21	39.93	36.04	33.13	30.73	28.66	26.78	25.03	23.37	21.76	20.22	18.77	17.38	(16.05)	(14.75)
11.00	∞	47.39	41.11	37.21	34.29	31.88	29.80	27.91	26.16	24.48	22.87	21.32	19.85	18.45	(17.11)	(15.79)
12.00	∞	48.47	42.18	38.27	35.35	32.93	30.85	28.95	27.19	25.51	23.89	22.33	20.86	19.45	(18.10)	(16.78)
13.00	∞	49.46	43.17	39.26	36.33	33.91	31.82	29.92	28.16	26.47	24.84	23.28	21.80	20.39	(19.03)	(17.71)
14.00	∞	50.38	44.08	40.17	37.24	34.82	32.73	30.82	29.06	27.36	25.73	24.17	22.69	21.28	(19.92)	(18.59)
15.00	∞	51.24	44.94	41.02	38.09	35.67	33.57	31.67	29.90	28.21	26.58	25.01	23.52	22.11	(20.75)	(19.42)
16.00	∞	52.04	45.74	41.82	38.89	36.46	34.37	32.47	30.70	29.00	27.37	25.80	24.32	22.90	(21.54)	(20.21)
17.00	∞	52.80	46.50	42.58	39.64	37.22	35.12	33.22	31.45	29.75	28.12	26.56	25.07	23.65	(22.29)	(20.96)
18.00	∞	53.51	47.21	43.29	40.36	37.93	35.84	33.93	32.16	30.47	28.83	27.27	25.77	24.37	(23.01)	(21.68)
19.00	∞	54.19	47.88	43.96	41.03	38.61	36.51	34.61	32.84	31.15	29.51	27.95	26.46	25.05	(23.69)	(22.36)
20.00	∞	54.83	48.52	44.61	41.67	39.25	37.16	35.26	33.49	31.79	30.16	28.60	27.11	25.70	(24.34)	(23.01)

TABLE 10. THE INTERNAL ENERGY $U - U_0$ (J MOLE⁻¹)

T (°K)	reduced density															
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
2.50	90.68	—	—	—	—	—	—	—	—	—	—	14.52	14.84	17.34	22.29	3.0
2.75	93.80	—	—	—	—	—	—	—	—	—	—	16.35	16.34	18.60	23.43	—
3.00	96.91	—	—	—	—	—	—	—	—	—	—	18.18	17.89	19.95	24.65	—
3.25	100.0	—	—	—	—	—	—	—	—	—	—	20.06	19.52	21.40	25.96	33.62
3.50	103.2	—	—	—	—	—	—	—	—	—	24.86	22.00	21.25	22.93	27.35	34.90
3.75	106.3	—	—	—	—	—	—	—	—	—	27.09	24.02	23.07	24.56	28.83	36.25
4.00	109.4	—	—	—	—	—	—	—	—	—	29.36	26.11	24.96	26.28	30.39	37.67
4.25	112.5	101.1	—	—	—	—	—	—	—	—	31.69	28.26	26.94	28.08	32.02	39.16
4.50	115.6	104.3	—	—	—	—	—	—	—	—	34.05	30.47	28.98	29.96	33.73	40.72
4.75	118.7	107.5	96.62	—	—	—	—	—	—	39.14	36.45	32.74	31.09	31.90	35.52	42.35
5.00	121.9	110.6	99.87	—	—	—	—	—	50.46	44.20	38.90	35.05	33.25	33.91	37.37	44.04
5.25	125.0	113.8	103.1	92.71	83.24	74.73	67.10	59.97	53.13	46.77	41.37	37.40	35.47	35.98	39.28	(45.79)
5.50	128.1	116.9	106.3	95.96	86.54	78.00	70.15	62.80	55.81	49.37	43.88	39.80	37.73	38.11	41.26	(47.61)
6.00	134.3	123.1	112.5	102.3	92.90	84.18	76.06	68.41	61.23	54.64	48.98	44.70	42.40	42.52	45.39	(51.43)
6.50	140.6	129.3	118.7	108.5	99.05	90.19	81.91	74.09	66.75	60.01	54.20	49.74	47.23	47.12	49.73	(55.48)
7.00	146.8	135.5	124.9	114.7	105.1	96.17	87.77	79.82	72.35	65.48	59.52	54.90	52.21	51.90	54.27	(59.75)
7.50	153.0	141.7	131.0	120.8	111.1	102.2	93.65	85.60	78.01	71.02	64.92	60.16	57.31	56.82	58.99	(64.22)
8.00	159.3	147.9	137.2	126.9	117.2	108.1	99.55	91.41	83.72	76.62	70.40	65.51	62.53	61.88	63.87	(68.87)
9.00	171.7	160.3	149.5	139.1	129.3	120.1	111.4	103.2	95.26	87.97	81.56	76.46	73.25	72.35	74.04	(78.70)
10.00	184.2	172.7	161.8	151.3	141.4	132.2	123.3	115.0	106.9	99.50	92.95	87.69	84.30	83.22	(84.71)	(89.12)
11.00	196.7	185.1	174.2	163.6	153.6	144.2	135.3	126.8	118.7	111.2	104.5	99.15	95.65	94.44	(95.79)	(100.0)
12.00	209.2	197.5	186.5	175.9	165.8	156.4	147.4	138.8	130.6	123.1	116.3	110.8	107.3	106.0	(107.2)	(111.3)
13.00	221.6	209.9	198.8	188.1	178.0	168.6	159.5	150.9	142.6	135.0	128.2	122.7	119.1	117.7	(118.9)	(123.0)
14.00	234.1	222.3	211.2	200.4	190.3	180.8	171.7	163.1	154.8	147.1	140.2	134.7	131.1	(129.7)	(130.8)	(134.8)
15.00	246.6	234.7	223.6	212.8	202.6	193.1	184.0	175.4	167.0	159.3	152.4	146.9	143.2	(141.8)	(142.9)	(146.9)
16.00	259.0	247.2	236.0	225.2	215.0	205.5	196.3	187.7	179.3	171.6	164.7	159.1	155.5	(154.0)	(155.1)	(159.1)
17.00	271.5	259.6	248.5	237.6	227.4	217.9	208.7	200.1	191.7	184.0	177.1	171.5	(167.8)	(166.4)	(167.5)	(171.5)
18.00	284.0	272.1	260.9	250.1	239.9	230.3	221.2	212.6	204.2	196.5	189.6	184.0	(180.3)	(178.9)	(180.0)	(184.0)
19.00	296.5	284.6	273.4	262.6	252.4	242.9	233.7	225.1	216.7	209.1	202.1	196.6	(192.9)	(191.5)	(192.6)	(196.7)
20.00	308.9	297.1	285.9	275.1	264.9	255.4	246.3	237.7	229.4	221.7	214.8	(209.3)	(205.6)	(204.2)	(205.3)	(209.4)

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TABLE 11. THE ENTHALPY $H - H_0$ (J MOLE⁻¹)

T (°K)	pressure (atm)																					
	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	
3.00	121.9	31.8	43.5	54.9	66.1	77.2	88.1	98.9	109.4	119.9	130.2	140.3	150.4	160.4	170.4	180.3	190.2	—	—	—	—	—
3.25	127.1	33.8	45.4	56.7	67.8	78.8	89.7	100.4	110.9	121.3	131.6	141.7	151.7	161.7	171.7	181.6	191.5	201.4	211.3	—	—	—
3.50	132.0	36.1	47.5	58.7	69.7	80.6	91.4	102.0	112.5	122.9	133.1	143.2	153.2	163.2	173.1	183.0	192.9	202.8	212.6	222.4	—	—
3.75	137.4	38.7	49.9	60.9	71.7	82.6	93.3	103.8	114.3	124.6	134.8	144.8	154.8	164.8	174.7	184.6	194.4	204.2	214.0	223.8	233.6	—
4.00	142.6	41.6	52.5	63.3	74.0	84.7	95.3	105.8	116.2	126.4	136.6	146.6	156.6	166.5	176.4	186.2	196.0	205.8	215.5	225.3	235.1	—
4.25	147.8	44.9	55.4	65.9	76.4	86.9	97.5	107.9	118.2	128.4	138.5	148.5	158.4	168.3	178.1	187.9	197.7	207.5	217.2	226.9	236.6	—
4.50	153.0	48.7	58.5	68.6	78.9	89.3	99.8	110.1	120.4	130.5	140.6	150.5	160.4	170.2	180.0	189.7	199.5	209.3	218.9	228.6	238.2	—
4.75	158.2	53.0	61.9	71.6	81.7	91.9	102.2	112.5	122.7	132.7	142.7	152.6	162.5	172.2	182.0	191.7	201.4	211.1	220.7	230.3	239.9	—
5.00	163.4	57.8	65.5	74.8	84.7	94.7	104.8	115.0	125.1	135.1	145.0	154.9	164.7	174.4	184.1	193.8	203.4	213.0	222.6	232.2	241.7	—
5.25	168.6	63.3	69.3	78.2	87.8	97.6	107.6	117.6	127.6	137.6	147.5	157.3	167.0	176.6	186.3	195.9	205.5	215.1	224.6	234.1	243.6	—
5.50	173.8	69.8	73.4	81.9	91.1	100.7	110.5	120.4	130.3	140.2	150.0	159.8	169.4	179.0	188.6	198.2	207.7	217.2	226.7	236.2	245.6	—
6.00	184.2	86.0	82.5	89.8	98.2	107.3	116.7	126.3	136.0	145.7	155.4	165.1	174.6	184.1	193.6	203.0	212.4	221.8	231.2	240.6	249.9	—
6.50	194.6	109.5	93.2	98.5	106.2	114.6	123.5	132.7	142.2	151.7	161.3	170.8	180.2	189.6	198.9	208.2	217.5	226.8	236.1	245.4	254.6	—
7.00	205.0	135.6	105.9	108.2	114.9	122.5	130.8	139.7	148.9	158.2	167.6	176.9	186.2	195.4	204.6	213.8	223.0	232.2	241.4	250.6	259.7	—
7.50	215.4	157.3	120.2	118.8	124.2	131.0	138.8	147.2	156.0	165.1	174.3	183.4	192.5	201.6	210.7	219.8	228.9	238.0	247.1	256.2	265.2	—
8.00	225.8	174.9	135.8	130.4	134.2	140.1	147.4	155.3	163.7	172.5	181.4	190.3	199.3	208.3	217.2	226.2	235.2	244.2	253.2	262.2	271.2	—
9.00	246.6	206.3	169.1	156.7	156.4	160.1	166.1	173.1	180.6	188.6	196.9	205.4	214.1	222.8	231.5	240.2	248.9	257.7	266.5	275.3	284.2	—
10.00	267.4	233.5	201.7	185.6	181.1	182.3	186.7	192.7	199.3	206.4	214.0	222.0	230.2	238.5	246.9	255.3	263.8	272.4	280.9	289.5	298.1	—
11.00	288.1	258.7	231.5	214.6	207.4	206.3	208.9	213.5	219.2	225.5	232.6	240.0	247.8	255.6	263.5	271.6	279.8	288.1	296.4	304.7	313.1	—
12.00	308.9	282.7	259.3	243.0	234.5	231.4	232.5	235.7	240.4	245.9	252.4	259.2	266.3	273.7	281.2	288.9	296.8	304.8	312.8	320.9	329.0	—
13.00	329.7	306.1	285.7	270.5	261.6	257.4	257.2	259.2	262.6	267.4	273.1	279.4	286.0	292.8	299.9	307.2	314.7	322.4	330.2	338.0	345.8	—
14.00	350.5	329.2	311.1	297.1	288.3	283.7	282.3	283.4	285.8	289.7	294.7	300.4	306.5	312.9	319.6	326.5	333.6	340.9	348.4	356.0	363.6	—
15.00	371.3	352.1	335.8	323.0	314.5	309.8	307.7	308.0	309.7	312.8	317.0	322.1	327.7	333.7	340.0	346.6	353.4	360.4	367.5	374.8	382.2	—
16.00	392.1	374.8	360.0	348.2	340.2	335.4	333.0	332.8	333.9	336.4	339.9	344.4	349.6	355.1	361.0	367.3	373.8	380.5	387.4	394.3	401.5	—
17.00	412.9	397.2	383.8	372.7	365.2	360.6	358.2	357.6	358.3	360.4	363.4	367.4	372.0	377.1	382.7	388.6	394.8	401.2	407.8	414.5	421.3	—
18.00	433.6	419.5	407.3	396.9	389.7	385.4	383.1	382.3	382.8	384.6	387.3	390.7	394.9	399.6	404.8	410.4	416.3	422.4	428.6	435.0	441.6	—
19.00	454.4	441.6	430.5	420.9	413.8	409.7	407.6	406.8	407.3	409.0	411.5	414.6	418.3	422.6	427.4	432.7	438.2	444.0	449.9	456.0	462.3	—
20.00	475.2	463.6	453.5	444.7	437.7	433.6	431.8	431.2	431.7	433.4	435.8	438.7	442.2	446.2	450.5	455.3	460.4	465.8	471.4	477.2	483.3	—

TABLE 12. THE FREE ENERGY $F - F_0$ (J MOLE⁻¹)

T (°K)	reduced density															
	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
2.50	∞	—	—	—	—	—	—	—	—	—	—	-4.00	-2.17	+1.83	+8.37	3.0
2.75	∞	—	—	—	—	—	—	—	—	—	—	-5.94	-3.93	+0.22	+6.92	—
3.00	∞	—	—	—	—	—	—	—	—	—	—	-8.04	-5.85	-1.51	+5.36	—
3.25	∞	—	—	—	—	—	—	—	—	—	—	-10.30	-7.90	-3.35	+3.70	+13.76
3.50	∞	—	—	—	—	—	—	—	—	—	-13.68	-12.72	-10.08	-5.31	+1.94	+12.17
3.75	∞	—	—	—	—	—	—	—	—	—	-16.50	-15.28	-12.38	-7.39	+0.07	+10.50
4.00	∞	—	—	—	—	—	—	—	—	—	-19.48	-17.97	-14.81	-9.57	-1.89	+8.74
4.25	∞	-49.9	—	—	—	—	—	—	—	—	-22.6	-20.8	-17.35	-11.87	-3.96	+6.88
4.50	∞	-58.9	—	—	—	—	—	—	—	-27.0	-25.8	-23.7	-20.0	-14.27	-6.13	+4.94
4.75	∞	-68.1	-49.0	—	—	—	—	—	—	-30.7	-29.2	-26.8	-22.8	-16.78	-8.39	+2.91
5.00	∞	-77.4	-56.8	—	—	—	—	—	-35.8	-34.6	-32.7	-30.0	-25.7	-19.39	-10.74	+0.79
5.25	∞	-86.8	-64.6	-54.6	—	—	-41.6	—	-40.2	-38.6	-36.4	-33.3	-28.7	-22.1	-13.18	(-1.42)
5.50	∞	-96.4	-72.7	-61.7	-61.7	—	-46.5	-48.7	-44.7	-42.7	-40.2	-36.7	-31.8	-24.9	-15.72	(-3.71)
6.00	∞	-116.1	-89.3	-76.3	-76.3	-68.7	-63.4	-59.7	-54.1	-51.4	-48.1	-43.9	-38.3	-30.9	-21.1	(-8.54)
6.50	∞	-136.3	-106.3	-91.5	-91.5	-82.4	-76.0	-71.2	-63.9	-60.4	-56.4	-51.4	-45.2	-37.2	-26.8	(-13.68)
7.00	∞	-157.0	-123.9	-107.1	-107.1	-96.6	-89.0	-83.2	-74.2	-69.8	-65.1	-59.4	-52.5	-43.8	-32.9	(-19.14)
7.50	∞	-178.1	-141.9	-123.1	-123.1	-111.2	-102.4	-95.7	-84.8	-79.7	-74.2	-67.8	-60.2	-50.8	-39.3	(-24.9)
8.00	∞	-199.6	-160.3	-139.6	-139.6	-126.2	-116.3	-108.5	-95.9	-89.9	-83.6	-76.5	-68.2	-58.1	-46.0	(-31.0)
9.00	∞	-243.8	-198.3	-173.7	-173.7	-157.4	-145.0	-135.2	-119.0	-111.4	-103.4	-94.9	-85.2	-73.8	-60.4	(-44.1)
10.00	∞	-289.4	-237.5	-209.1	-209.1	-189.9	-175.1	-163.3	-143.4	-134.2	-124.6	-114.5	-103.4	-90.6	-75.8	(-58.3)
11.00	∞	-336.2	-278.0	-245.7	-245.7	-223.6	-206.4	-192.5	-169.0	-158.1	-147.0	-135.3	-122.7	-108.6	-92.4	(-73.6)
12.00	∞	-384.1	-319.7	-283.4	-283.4	-258.4	-238.8	-222.8	-195.7	-183.1	-170.4	-157.2	-143.0	-127.5	-110.0	(-89.9)
13.00	∞	-433.1	-362.4	-322.2	-322.2	-294.3	-272.2	-254.2	-223.4	-209.1	-194.7	-180.0	-164.3	-147.4	-128.6	(-107.2)
14.00	∞	-483.0	-406.0	-361.9	-361.9	-331.1	-306.6	-286.5	-252.0	-236.0	-220.0	-203.7	-186.6	-168.2	-148.1	(-125.3)
15.00	∞	-533.9	-450.5	-402.5	-402.5	-368.8	-341.9	-319.6	-281.5	-263.8	-246.2	-228.3	-209.7	-189.9	-168.4	(-144.3)
16.00	∞	-585.5	-495.8	-443.9	-443.9	-407.2	-377.9	-353.6	-311.9	-292.4	-273.2	-253.7	-233.6	-212.4	-189.5	(-164.1)
17.00	∞	-638.0	-542.0	-486.2	-486.2	-446.5	-414.8	-388.3	-343.0	-321.8	-300.9	-280.0	-258.3	-235.7	-211.4	(-184.7)
18.00	∞	-691.1	-588.9	-529.1	-529.1	-486.5	-452.4	-423.8	-374.8	-352.0	-329.4	-306.9	-283.7	-259.8	-234.1	(-206.0)
19.00	∞	-745.0	-636.4	-572.7	-572.7	-527.2	-490.7	-460.0	-407.3	-382.8	-358.6	-334.5	-309.8	-284.5	-257.5	(-228.0)
20.00	∞	-799.5	-684.5	-617.1	-617.1	-568.5	-529.6	-496.9	-440.4	-414.1	-388.4	-362.7	-336.6	-309.8	-281.5	(-250.6)

THERMODYNAMIC PROPERTIES OF FLUID HELIUM

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TABLE 13. THE SPECIFIC HEAT C_p (J DEG⁻¹ MOLE⁻¹)
pressure (atm)

T (°K)	0	5	10	15	20	30	40	50	60	70	80	90	100
3.00	20.8	8.71	7.68	7.11	6.72	6.17	5.81	5.55	5.35	5.20	5.08	—	—
3.25	20.8	9.74	8.43	7.79	7.35	6.73	6.32	6.02	5.79	5.61	5.46	5.34	—
3.50	20.8	10.8	9.21	8.50	8.00	7.31	6.84	6.50	6.24	6.03	5.85	5.71	5.56
3.75	20.8	11.8	10.0	9.21	8.65	7.89	7.37	6.99	6.70	6.46	6.25	6.08	5.93
4.00	20.8	12.9	10.8	9.90	9.29	8.47	7.90	7.49	7.17	6.90	6.67	6.47	6.30
4.25	20.8	14.0	11.7	10.6	9.93	9.04	8.43	7.99	7.63	7.34	7.09	6.87	6.67
4.50	20.8	15.2	12.5	11.4	10.6	9.60	8.96	8.47	8.09	7.78	7.51	7.27	7.05
4.75	20.8	16.4	13.4	12.1	11.3	10.2	9.48	8.95	8.54	8.21	7.92	7.67	7.44
5.00	20.8	18.9	14.5	12.9	12.0	10.8	9.99	9.42	8.99	8.64	8.34	8.07	7.83
5.25	20.8	22.1	15.7	13.7	12.6	11.3	10.5	9.88	9.44	9.07	8.75	8.48	8.23
5.50	20.8	24.9	17.0	14.5	13.3	11.9	11.0	10.4	9.89	9.49	9.17	8.89	8.63
6.00	20.8	34.0	19.9	16.2	14.7	13.0	12.0	11.3	10.8	10.3	10.0	9.71	9.43
6.50	20.8	56.0	22.9	18.1	16.1	14.1	13.0	12.2	11.6	11.1	10.8	10.5	10.2
7.00	20.8	46.3	26.0	20.1	17.6	15.2	13.9	13.1	12.4	11.9	11.5	11.2	10.9
8.00	20.8	33.2	32.1	23.9	20.5	17.3	15.8	14.7	13.9	13.4	12.9	12.6	12.2
9.00	20.8	28.2	33.1	27.0	23.1	19.3	17.5	16.3	15.4	14.8	14.2	13.9	13.5
10.00	20.8	25.9	30.5	28.6	25.2	21.0	19.0	17.7	16.7	16.1	15.4	15.0	14.7
11.00	20.8	24.7	28.0	28.5	26.4	22.4	20.3	18.9	17.9	17.2	16.6	16.1	15.8
12.00	20.8	23.9	26.5	27.7	26.8	23.5	21.4	20.0	19.0	18.2	17.6	17.1	16.7
14.00	20.8	23.0	24.7	25.7	26.0	24.7	23.1	21.8	20.7	19.9	19.3	18.7	18.3
16.00	20.8	22.5	23.6	24.5	25.1	25.0	24.1	23.0	22.0	21.2	20.6	20.0	19.5
18.00	20.8	22.2	23.0	23.7	24.2	24.6	24.4	23.7	22.9	22.2	21.6	21.0	20.5
20.00	20.8	21.9	22.6	23.2	23.7	24.2	24.2	23.9	23.6	23.0	22.3	21.8	21.3